When Portfolio Choice Meets Hat Algebra: 
An Integrated Approach to International Finance and Trade

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Abstract

This paper develops an approach for solving a multi-country model where financial and trade flows are both endogenously determined. The approach combines the perturbation method for portfolio choice analysis in a DSGE framework and the hat algebra technique from the trade literature. Comparative statics employing this approach captures the interaction of financial and trade channels in the general equilibrium, which is not fully characterized by standard international trade or macro models. Policy experiments based on the model quantify the impacts of tariffs and financial frictions as barriers to globalization.

JEL Codes: F10, F41, F60

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1 Introduction

Cross-country commodity and capital flows serve as two paramount engines of globalization. Nonetheless, few models have been proposed to characterize both trade and financial linkages in a multi-country structural framework. This paper develops an approach for solving a general equilibrium model where trade and finance interact. Questions that can be answered by this new approach include how a trade war reshapes the pattern of international capital allocation, and how regional financial integration influences the direction and volume of global trade flows.

The approach not only combines the recent breakthroughs from both international macro and trade literatures, but also mitigates the methodological challenges faced by each strand and yields different predictions from existing works. Compared to the trade literature that typically takes countries’ asset positions as exogenous, this paper solves global financial allocation under agents’ intertemporal utility maximization decisions and cross-country frictions. Compared to the international macro literature that usually studies a small number of countries, this paper examines the multilateral linkages across many economies of uneven sizes. Specifically, I embed portfolio choice analysis in a quantitative trade model, and examine 43 calibrated economies linked through trade and financial exchanges. The endogenous portfolio captures agents’ risk-sharing motives influenced by the global trade pattern. Meanwhile, countries’ asset positions shift the world demand system in the commodity market. Therefore, this approach permits a higher degree of interplay between trade and financial channels in the general equilibrium, which is difficult to be fully captured by standard international macro or trade models.

To illustrate the main idea of the approach, I develop a model that builds on the Eaton and Kortum (2002) trade framework with immobile human and physical capital endowments and a single tradable sector subject to iceberg trade costs. Besides intermediate goods, financial assets which are claims to countries’ capital income are traded across countries. Financial frictions that vary across country pairs add costs households’ repatriation of foreign returns. Nonetheless, households hold different assets to reduce the impact of country-specific productivity shocks on their consumption through international risk sharing. To derive asset positions, I follow Devereux and Sutherland (2011)’s method which solves portfolios in open economy macro models.\(^1\) It combines a second-

\(^1\)Devereux and Sutherland (2011)’s method can be used to analyze multiple types of assets (e.g. equities or bonds) under different financial structures (complete or incomplete markets) in a DSGE model of any dimension solvable by the local approximation technique.
order approximation of agents’ Euler equation with a first-order approximation of other model equations to determine a steady-state portfolio decided by agents’ forward-looking utility maximization decisions. The method is flexible enough to be applied to a wide range of DSGE frameworks, but it is computationally challenging to implement when the number of variables rises substantially with the number of countries. 

I employ the ‘exact hat algebra’ technique developed by Dekle et al. (2007) from the trade literature to overcome this computational challenge. This tractable technique enables comparative static analyses for policy experiments in a rich environment with numerous economies of uneven sizes connected through multilateral economic linkages. In response to changes in trade or financial frictions, the total changes of any variable may include 1) the change of its steady-state value under different policy regimes (inter-regime changes), and 2) the deviation of the variable from its steady state under shocks within a specific policy regime (intra-regime changes). I use the hat algebra method to characterize both types of changes: one globally to measure the distance between steady states across original and counterfactual regimes, and the other locally within each regime around its steady state. Countries’ asset positions, determined by intra-regime changes of second-moment variables in response to simulated shocks around a steady state, will simultaneously influence inter-regime changes by shifting countries’ expenditure. The counterfactual outcome is obtained as the solution to a joint fixed-point problem of changes to wage on the real side and to portfolio on the financial side. By characterizing both the size and composition of countries’ financial allocation in the general equilibrium, this approach makes an important contribution to the trade literature, which typically treats financial flows either as exogenously determined with values taken from the data or endogenously derived under extreme circumstances such as financial autarky or complete markets.

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2This is because the local approximation method typically requires the computation of steady-state values of all the variables and the loglinearization of all the equations that characterize the variables’ relations, which is difficult to implement when there are numerous countries with sparse bilateral ties. This is a common challenge for international macro models, which normally have to limit their inquiry scope to the analysis of a small open economy or two symmetric economies (for example, canonical models developed by Mendoza (1991) and Backus et al. (1992)). Such a framework cannot capture all the cross-country bilateral linkages and multilateral resistances in the spirit of Anderson and Van Wincoop (2003).

3The hat algebra technique is tractable not only because it adopts an iterative procedure that solves numerical solutions almost instantaneously, but also because it does not require knowledge of all the variables’ levels except a few observable statistics to predict counterfactuals.

4For example, Dekle et al. (2007) and Alvarez (2017) examine counterfactual trade patterns under financial autarky. Eaton et al. (2016) study the puzzles in international macroeconomics under complete markets. It is challenging to analyze general cases between these two extreme financial arrangements due to the difficulty of solving the portfolio choice problem in a multilateral general equilibrium model.
I conduct policy experiments based on the calibrated model under alternative financial frictions and tariffs to illustrate the impacts of barriers to globalization. Given higher cross-country financial frictions, countries reduce asset positions under greater barriers to global investment. Nevertheless, they turn out to adjust portfolios less to financial frictions when assets' covariance structure offers risk-sharing benefits. Meanwhile, given higher cross-country tariffs, most countries witness greater declines to real wage yet increases to asset holdings when financial positions are adjustable. This happens because higher tariffs impair international output synchronization, which reduces cross-country asset covariances in the financial market. The increased financial holdings allow countries to raise expenditure, therefore their welfare loss would be overestimated if endogenous financial adjustments in response to the real side of the economy were not considered. This comparison between fixed and adjustable asset positions highlights the importance of incorporating a financial channel in quantitative trade models.

Furthermore, to compare this multi-country framework with a standard two-country framework in the international macro literature, I examine the model predictions from a scenario where every country sees itself as the domestic economy and all the other countries as the foreign economy when re-constructing and re-calibrating a two-economy model for each country. The results under higher tariffs suggest that most countries will increase their asset holdings by a greater magnitude in the multi-economy than in the two-economy case, largely due to the diversification benefits offered by all the cross-country asset covariances which are not fully captured in the two-economy setup. Hence, this comparison highlights the importance of embedding multilateral linkages across countries in open economy macro models.

Last but not least, I study the recent China-US trade war as an application of the model. Under higher tariffs between the two economies, the model predicts that most countries suffer a decline in real wage due to the rise in commodity prices, yet they raise asset holdings which partially offset the welfare shortfall. Moreover, I examine a counterfactual scenario where China sells off its holding of US assets and reconstructs the portfolio among remaining countries under the asset covariance structure. The model predicts that, the welfare of China will decrease by another 1.3 percent if this financial retaliation occurs. Therefore, the decoupling of the two major economies in trade and financial channels is very likely to bring more costs than benefits to themselves and to the world economy.

This paper contributes to both international macro and trade literatures. The portfolio choice analysis employs the method developed by Devereux and Sutherland (2011)
who use a second-order approximation to overcome the certainty equivalent in a first-order approximation. Similar insights are also formulated by Samuelson (1970), Judd and Guu (2001), and Tille and Van Wincoop (2010). The method offers a powerful tool in international macroeconomics to solve perplexing puzzles like asset home bias (see, for example, Coeurdacier and Rey (2013), Coeurdacier and Gourinchas (2016), and Hu (2020)). Compared to the portfolio techniques driven by investors’ specific preference for assets, such as the asset demand system employed by Liu et al. (2022) and the rational inattention logit demand system adopted by Pellegrino et al. (2021), this method does not require separate utility assumptions for agents’ intratemporal financial allocation, which is determined by the endogenous asset covariance structure instead. Plus, the method is capable of capturing the general equilibrium effects of macro and financial variables, thanks to its compatibility with open economy macro models. However, the downside of this method is that portfolios usually cannot be expressed in an analytical form, especially in the presence of financial frictions. Besides, it is computationally difficult to obtain numerical solutions when applied to many economies with sparse bilateral linkages, but the difficulty is alleviated when I use hat algebra for comparative statics.

This paper contributes to the trade literature by modeling cross-country financial flows endogenously. Focused on the real side of the economy, most trade models are static and ignore intertemporal decisions. The recent exceptions which are closer to this work are Alvarez (2017) and Kleinman et al. (2021) who introduce forward-looking physical capital accumulation decisions. Nevertheless, they follow the standard trade approach by assuming balanced trade or taking the asset positions as exogenous from data. Focusing on financial assets instead of physical capital, my approach generates predictions for how both the size and composition of a country’s asset position fluctuate given households’ forward-looking investment decisions. Plus, I derive counterfactual financial linkages with the hat algebra technique (Dekle et al. (2007)), which has been commonly used in quantitative spatial models (see Redding and Rossi-Hansberg (2017) for a survey). Adding financial flows, which exhibit similar geographic patterns to trade and migration flows according to Portes and Rey (2005), completes the spatial analysis.

The remainder of the paper proceeds as follows: Section 2 develops a multi-country model with trade and financial linkages, and describes the method employed to solve the model. Section 3 conducts policy experiments to explain the economic mechanisms and to highlight the differences of model predictions made by this framework and by existing literature. Section 4 examines the China-US trade war as an application of the model. Section 5 concludes.
2 Theory

2.1 Model Setup

The world comprises $I$ countries indexed $i = 1, ..., I$. Each of them produces a final composite good using a continuum of intermediate goods $u \in [0, 1]$ traded across countries

$$Q_{i,t} = \int_0^1 [q_{i,t}(u)]^{1-\epsilon} \, du, \quad (1)$$

where $\epsilon$ is the elasticity of substitution in the CES aggregator. The composite good can be used either for consumption $C_{i,t}$ or for the production of intermediate goods in country $i$. Country $i$ draws productivity $Z_{i,t}(u)$ when producing $u$ from a Frechet distribution

$$F_{i,t}(z) = \exp(-T_{i,t}z^{-\theta}). \quad (2)$$

To characterize the risks of the economy for portfolio analysis, I assume that country-specific productivity $T_{i,t}$ is subject to serially independent shocks $\epsilon_{i,t}$ drawn from a joint normal distribution with a variance-covariance matrix $\Sigma_T$ that contains $\text{var}(\epsilon_{i,t}^2) = \sigma_i^2$, $\text{cov}(\epsilon_{i,t}, \epsilon_{j,t}) = \sigma_{ij}$, $\forall i, j \in [1, I]$ around its mean value over time $\bar{T}_t$.\footnote{This assumption about productivity shocks is in the same spirit as the international real business cycle literature that typically examines the responses of macro variables to country-specific productivity shocks (see Mendoza (1991), Backus et al. (1992), and Aguiar and Gopinath (2007) among many others). Nevertheless, I am agnostic about the nature of risks that drive countries' output fluctuations. Besides productivity shocks, this model can be adapted to accommodate other shocks which can also induce agents to construct portfolios for cross-country risk sharing.}

Production of intermediate goods combines country-specific labor and capital endowments denoted as $L_i$ and $K_i$ which are assumed to be fixed in supply.\footnote{To deliver the main idea of the method, I assume physical capital is a fixed endowment for the tractability of this simple model and focus on financial capital as means of consumption smoothing and international risk sharing. It is possible in future extensions to incorporate physical capital accumulation following Alvarez (2017) and Kleinman et al. (2021).} Moreover, it uses country $i$’s composite good as an input for production. Let $w_{i,t}$, $r_{i,t}$, and $P_{i,t}$ be the prices of these inputs, and let $\tau_{ij}$ be the iceberg trade cost for goods exported from country $i$ to $j$, country $i$’s cost of serving a good $u$ to country $j$ at time $t$ is hence given by

$$p_{ij,t}(u) = \frac{\tau_{ij}[(r_{i,t}^{1-\mu}w_{i,t}^{1-\mu})^{\eta}P_{i,t}^{1-\eta}]}{Z_{i,t}(u)}, \quad (3)$$

where $\mu$ is the share of capital and $1-\eta$ is the share of the composite good in production.
It follows that the share of $i$’s goods in $j$’s expenditure is

$$\pi_{ij,t} = \frac{T_{i,t}[\tau_{ij}(r^\mu_{it}w^{1-\mu}_{i,t})^\eta P_{i,t}^{1-\eta}]^{-\theta}}{\Phi_{j,t}},$$

where

$$\Phi_{j,t} = \sum_{k=1}^I T_{k,t}[\tau_{kj}(r^\mu_{kt}w^{1-\mu}_{k,t})^\eta P_{k,t}^{1-\eta}]^{-\theta},$$

(4)

while $\Phi_{j,t}$ is linked to the price level in country $j$ through

$$P_{j,t} = \Gamma \Phi_{j,t}^{-\frac{1}{\theta}},$$

(5)

where $\Gamma$ represents a Gamma function: $\Gamma(\frac{1-\epsilon}{\theta} + 1)^{\frac{1}{1-\epsilon}}$.

The goods market clearing condition of country $i$ thus follows

$$Y_{i,t} = \sum_{j=1}^I \pi_{ij,t} X_{j,t},$$

(6)

where $X_{j,t}$ is country $j$’s expenditure and $Y_{i,t}$ is $i$’s output. $X_{i,t}$ and $Y_{i,t}$ are also connected through $D_{i,t}$ which is country $i$’s aggregate asset position

$$X_{i,t} = Y_{i,t} D_{i,t}.$$  

(7)

On the financial side, I follow the international macro literature including Coeurdacier and Rey (2013) to assume countries issue one-period equities whose dividends are claims to their capital income:

$$d_{i,t} = \mu Y_{i,t},$$

(8)

which together with equity prices $p_{E,i,t}$ define financial returns

$$R_{i,t+1} = \frac{d_{i,t+1} + p_{E,i,t+1}}{p_{E,i,t}}.$$  

(9)

Households hold assets to maximize expected lifetime utility

$$\max \sum_{t=0}^\infty \beta^t \frac{C_{i,t}^{1-\gamma}}{1-\gamma},$$

(10)

where $\beta$ is the discount factor and $\gamma$ is the coefficient of relative risk aversion in the
CRRA utility function, both of which appear in the intertemporal Euler equation
\[ \frac{C_{i,t}^{-\gamma}}{P_{i,t}} = \beta E_t \left[ \frac{C_{i,t+1}^{-\gamma}}{P_{i,t+1}} e^{-f_{ij} R_{j,t+1}} \right], \quad \forall i, j \in [1, I]. \] (11)

Markets are incomplete due to the existence of barriers to global financial investment. In particular, financial frictions potentially vary across country pairs, which justifies the gravity model of capital flows documented by Portes and Rey (2005). I follow Heathcote and Perri (2004) and Aviat and Coeurdacier (2007) by introducing financial frictions as iceberg transaction costs \( f_{ij} \geq 0 \), so that households in country \( i \) expect to collect \( e^{-f_{ij}} R_{j,t+1} \) when repatriating asset returns from country \( j \). Besides, these frictions are second-order in magnitude (proportional to the variance of shocks in the model) so that I can employ the solution method for portfolio choice in an open economy DSGE framework developed by Devereux and Sutherland (2011). Acknowledging that assets are distinguishable by their risk characteristics, these authors develop a method that combines a second-order approximation of the portfolio choice equation derived from the Euler equation (11) with a first-order approximation of other equations of the model in order to determine a zero-order (i.e. steady-state) portfolio.

Country \( i \)'s portfolio choice equation and wealth constraint are given by
\[ E_t \left[ \frac{C_{i,t}^{-\gamma}}{P_{i,t}} R_{i,t+1} \right] = E_t \left[ \frac{C_{i,t+1}^{-\gamma}}{P_{i,t+1}} e^{-f_{ij} R_{j,t+1}} \right], \quad \forall i, j \in [1, I], \] (12)
\[ W_{i,t+1} = W_{i,t} R_{I,t+1} + \sum_{j=1}^{I-1} \alpha_{ij} (R_{j,t+1} - R_{I,t+1}) + \eta Y_{i,t} - P_{i,t} C_{i,t}. \] (13)

In equation 13, \( \alpha_{ij} \) is \( i \)'s holding of \( j \)'s assets in the steady state of the economy and \( W_{i,t} \) is country \( i \)'s net asset position at \( t \). Country \( I \)'s asset is assumed to be a numeraire and \( R_{j,t+1} - R_{I,t+1} \) reflects the excess return of other assets relative to it. It is worth noting that portfolio choice derived from the Euler equations (11 and 12) captures both inter-temporal and intra-temporal investment decisions of households to maximize their expected lifetime utility (10). Inter-temporally, households decide between financial investment and current financial frictions makes the portfolio choice problem tractable. However, I do not take a strong stand on either the underlying structure or the theoretical foundation of these barriers to international financial investment. Specifically, the bilateral friction \( f_{ij} \) can reflect a mix of worldwide factors including global financial liquidity, country-specific factors including capital account openness, and pair-specific factors including geographic distance and bilateral financial agreements. It can take alternative forms such as informational frictions, as Okawa and Van Wincoop (2012) find that these types of frictions yield comparable implications for the gravity model of financial flows.
consumption, given their patience ($\beta$) and elasticity of intertemporal substitution ($\gamma$), upon expected asset returns $R_{j,t+1}$ and inflation (dynamics of $P_{i,t+1}$). Intra-temporally, the covariance matrix of different countries’ productivity shocks ($\Sigma_T$) and the matrix of bilateral financial frictions will appear in the second-order Taylor expansion of the Euler equation 12, evaluating which determines portfolio choice. Therefore, households will naturally prefer assets from countries whose shocks are less correlated with their home country’s under risk-sharing motives, and whose assets are subject to lower transaction costs to maximize financial payoff.

To sum up the description of the model setup, the general equilibrium of the model consists of a set of prices and quantities such that 1) households choose consumption and construct portfolio to maximize expected lifetime utility, 2) firms set output and price to maximize profit, and 3) factor, commodity, and asset markets clear. The economy is deemed in a steady state when the endogenous variables that satisfy all the equilibrium conditions are constant over time.

2.2 Solution Techniques

Solving for the equilibrium portfolio choice in a DSGE framework with the perturbation method developed by Devereux and Sutherland (2011) requires log-linearizing the model around the steady state of the economy.\textsuperscript{8} Let $\tilde{A}_t$ represent the log-deviation of any variable $A$ from its steady state $\bar{A}$ at $t$

$$\tilde{A}_t = \ln\left(\frac{A_t - \bar{A}}{A}\right),$$

then the cross-country ratio of any country-specific variable $B_{i,t}$ defined as

$$B_{i,j,t} = \frac{B_{i,t}}{B_{j,t}},$$

has its deviation from the steady state expressed as

$$\tilde{B}_{i,j,t} = \tilde{B}_{i,t} - \tilde{B}_{j,t}.$$
I assume $I$ is a numeraire country when solving the world matrix of portfolio weights$^9$

$$
\alpha = \begin{bmatrix}
\alpha_{11} & \alpha_{12} & \cdots & \alpha_{1I-1} \\
\alpha_{21} & \ddots & & \vdots \\
\vdots & \ddots & \ddots & \vdots \\
\vdots & & \ddots & \alpha_{I-2I-1} \\
\alpha_{I-11} & \cdots & \alpha_{I-1I-2} & \alpha_{I-1I-1}
\end{bmatrix}, \quad (17)
$$

whose elements in the $i^{th}$ row are decided by country $i$'s Euler equation (12) which satisfies

$$
E_t\left[ \frac{U'(C_{i,t+1})}{P_{i,t+1}} e^{-f_i I} R_{I,t+1} \right] = E_t\left[ \frac{U'(C_{i,t+1})}{P_{i,t+1}} e^{-f_{I-1} I} R_{I-1,t+1} \right] = \cdots = E_t\left[ \frac{U'(C_{i,t+1})}{P_{i,t+1}} e^{-f_{I-1} I} R_{I-1,t+1} \right]. \quad (18)
$$

Portfolio is derived from the second-order Taylor expansion of equation 18 while taking the difference between the numeraire asset $I$ and all the other assets

$$
E_t[\tilde{R}_{x,t+1} + \frac{1}{2} \tilde{R}_{x,t+1}^2 - \tilde{R}_{x,t+1}(\gamma \tilde{C}_{i,t+1} + \tilde{P}_{i,t+1})] = -\frac{1}{2} F_i + O(\epsilon^3), \quad (19)
$$

where $R_{x,t+1}$ denotes a vector of excess returns relative to the numeraire asset

$$
\tilde{R}_{x,t+1} = [\tilde{R}_{1,t+1} - \tilde{R}_{I,t+1}, \tilde{R}_{2,t+1} - \tilde{R}_{I,t+1}, \ldots, \tilde{R}_{I-1,t+1} - \tilde{R}_{I,t+1}], \quad (20)
$$

$R_{x,t+1}^2$ denotes the vector of excess squared returns

$$
\tilde{R}_{x,t+1}^2 = [\tilde{R}_{1,t+1}^2 - \tilde{R}_{I,t+1}^2, \tilde{R}_{2,t+1}^2 - \tilde{R}_{I,t+1}^2, \ldots, \tilde{R}_{I-1,t+1}^2 - \tilde{R}_{I,t+1}^2], \quad (21)
$$

and $F_i$ denotes $i$'s vector of financial frictions defined as

$$
F_i' = [f_{i1} - f_{11}, f_{i2} - f_{22}, \ldots, f_{I1} - f_{I1}], \quad (22)
$$

whose $k^{th}$ element represents the additional financial friction country $i$'s households incur when holding $I$’s relative to $k$’s asset. $O(\epsilon^3)$ captures all terms of order higher than two.

The difference between any country $i$’s and the numeraire country $I$’s expanded Euler
Equations (19) follows

\[ E_t[(\gamma \tilde{C}_{i/I,t+1} + \tilde{P}_{i/I,t+1})\tilde{R}_{x,t+1}] = \frac{1}{2} F_{iI} + O(\epsilon^3), \quad \forall i \in [1, I-1], \]  

(23)

where \( F_{iI} \) stands for the excess financial frictions faced by country \( i \) relative to by \( I \)

\[ F_{iI} = F'_i - F'_I. \]  

(24)

Equation 23 is country \( i \)'s relative to \( I \)'s portfolio determination equation: the variables on its left hand side covary with country \( i \)'s asset positions which are also influenced by financial frictions \( F_{iI} \) on the right. When evaluating the equation to pin down the equilibrium portfolio, we consider the responses of \( \gamma \tilde{C}_{i/I,t+1} + \tilde{P}_{i/I,t+1} \) and \( \tilde{R}_{x,t+1} \) to all the productivity shocks. A standard DSGE approach would take the first-order derivative of the two variables with respect to the vector of productivity shocks, denoted as \( \tilde{G}_{t+1} \) and \( \tilde{H}_{t+1} \) respectively in

\[ E_t(\tilde{G}_{t+1} \times \Sigma_T \times \tilde{H}_{t+1}) = \frac{1}{2} F_{iI} + O(\epsilon^3). \]  

(25)

Nonetheless, solving the portfolio choice problem with a standard DSGE approach is challenging especially in a multi-country scenario, because it is a daunting task to compute all the steady-state values and loglinearize all the equations when the number of countries is large. I find that these challenges are significantly mitigated in a quantitative trade framework for two reasons. First, establishing the relationship among variables from a large number of countries in a closed form is easier. Second, applying the ‘hat algebra’ method developed by trade economists makes the computation for counterfactual results faster. Instead of taking a first-order perturbation to evaluate equation 25, I simulate productivity shocks and compute variables’ responses directly around the steady state of the economy.

\footnote{This makes it sufficient to focus on the variables that appear in the portfolio determination equation (26) only. Other variables in the model which influence portfolio choice can be expressed as analytical functions of those variables in the equation.}

\footnote{This is thanks to 1) the efficient iterative computation procedure, whose solution captures the dynamics of variables, replaces the need for loglinearizing all the equations in the model to predict variables’ responses to shocks, and 2) the fact that it is no longer necessary to solve for the steady-state values of all the variables; A few observable statistics are sufficient for policy experiments.}
To solve for portfolios using this new approach, I rewrite equation 23 as \[ E_t[(\gamma(1-\beta)\tilde{Y}_{i,t+1} + (1-\gamma+\beta\gamma)\tilde{P}_{i,t+1} + \gamma(1-\beta)(\tilde{\alpha}_i - \tilde{\alpha}_I)\tilde{R}_{x,t+1})\tilde{R}_{x,t+1}'] = \frac{1}{2}F_{it}, \] (26)

where portfolio weights are scaled by a holder country’s output and discount factor:

\[ \tilde{\alpha}_{ik} = \frac{\alpha_{ik}}{\beta Y_i}, \] (27)

which constitute the country’s vector of asset holdings

\[ \tilde{\alpha}_i = [\tilde{\alpha}_{i1}, \tilde{\alpha}_{i2}, ..., \tilde{\alpha}_{ii-1}]. \] (28)

Equation 26, if stacked vertically with each row representing a country, constructs a system of equations for the world bilateral portfolio matrix (defined in 17) to be solved.

Policy experiments under counterfactual trade and financial frictions are conducted to examine the impacts of these barriers to globalization. Given the counterfactual frictions, equation 26 is evaluated in original and counterfactual cases, where variables are superscripted \( s = \{\text{org, ctf}\} \) respectively:

\[ E_t[(\gamma(1-\beta)\tilde{Y}_{org,i,t+1} + (1-\gamma+\beta\gamma)\tilde{P}_{org,i,t+1} + \gamma(1-\beta)(\tilde{\alpha}_{org,i} - \tilde{\alpha}_I)\tilde{R}_{org,x,t+1})\tilde{R}_{org,x,t+1}'] = \frac{1}{2}F_{it}^{\text{org}}, \] (29)

\[ E_t[(\gamma(1-\beta)\tilde{Y}_{ctf,i,t+1} + (1-\gamma+\beta\gamma)\tilde{P}_{ctf,i,t+1} + \gamma(1-\beta)(\tilde{\alpha}_{ctf,i} - \tilde{\alpha}_I)\tilde{R}_{ctf,x,t+1})\tilde{R}_{ctf,x,t+1}'] = \frac{1}{2}F_{it}^{\text{ctf}}. \] (30)

If the numeraire country (I) does not adjust portfolio, the difference between equations 29 and 30 will decide changes in country \( i \)’s bilateral asset holdings \( \Delta \tilde{\alpha}_i = \tilde{\alpha}_{ctf,i} - \tilde{\alpha}_{org,i} \) in response to any change in the products (second moments) of variables that appear in the equations including \( \tilde{Y}_{s,i,t+1} \tilde{R}_{x,t+1}' \), \( \tilde{P}_{s,i,t+1} \tilde{R}_{x,t+1}' \), \( \tilde{R}_{s,x,t+1} \tilde{R}_{s,x,t+1}' \), \( s \in \{\text{org, ctf}\} \) (31)

as well as in financial frictions \( \Delta F_{it} = F_{it}^{ctf} - F_{it}^{org} \) across original and counterfactual scenarios. The resulting bilateral asset holdings will add up to countries’ equilibrium aggregate position as shares of output \( \tilde{D}_i = \beta \sum_{k=1}^{I} \tilde{\alpha}_{ik} \).

In policy experiments, counterfactual trade or financial frictions can affect both the level (first moment) and covariance (second moment) of variables in response to shocks.

\[ ^{12} \text{This step uses countries’ wealth constraint to substitute out consumption as a function of other variables solvable by hat algebra. See Appendix A for the detailed derivation.} \]
of the economy. The total dynamics of relative output of country $i$ to $j$ at time $t + 1$ under counterfactual versus original frictions can be decomposed into two parts

$$
\Delta Y_{i/j} = \Delta \hat{Y}_{i/j} + \Delta \tilde{Y}_{i/j},
$$

(32)

From this decomposition, the total changes of a variable reflect 1) the change of its steady-state value under different policy regimes of frictions (called inter-regime changes here), and 2) the deviation of the variable from its steady state under shocks within a specific policy regime (called intra-regime changes). Inter-regime changes are derived in a similar way as in trade models, while examining intra-regime changes of how variables behave around steady states is necessary to solve the equilibrium asset position. Now I describe how inter- and intra-regime changes can both be characterized by hat algebra.

Inter-regime changes are computed following Dekle et al. (2007)’s analysis where they introduced the hat algebra method to examine a scenario without global imbalances. Let the ratio of any variable $A$’s counterfactual to original steady-state value under policy changes be denoted as

$$
\hat{A} = \frac{\hat{A}_i^{ctf}}{\hat{A}_i^{org}}.
$$

(33)

It follows that the vectors of all the countries’ wage and price

$$
\hat{\bar{w}}' = [\hat{\bar{w}}_1, \hat{\bar{w}}_2, ..., \hat{\bar{w}}_I], \quad \hat{\bar{P}}' = [\hat{\bar{P}}_1, \hat{\bar{P}}_2, ..., \hat{\bar{P}}_I]
$$

(34)

given countries’ counterfactual asset positions

$$
\hat{\bar{D}}^{ctf} = [\hat{\bar{D}}^{ctf}_1, \hat{\bar{D}}^{ctf}_2, ..., \hat{\bar{D}}^{ctf}_I]
$$

(35)

are obtained by an iterative computation procedure to solve a fixed-point problem for a pair of vectors ($\hat{\bar{w}}, \hat{\bar{P}}$) based on all the countries’ price determination and goods market clearing conditions (see Appendix A for the derivation of equations):

$$
\hat{\bar{P}}_i^{-\theta} = \sum_{j=1}^{I} \frac{\hat{\bar{P}}_j^{org}}{\hat{\bar{P}}_j^{ctf}} \left( \frac{\hat{\bar{P}}_j^{ctf}}{\hat{\bar{P}}_j^{org}} \right)^{-\theta},
$$

(36)
\[
\hat{w}_i \hat{Y}_i^{org} = \sum_{j=1}^{I} \frac{\pi_{ij}^{org} \tau_{ij}^{\theta} (\hat{w}_i \hat{P}_i)^{1-\eta} - \theta}{\pi_{kj}^{org} \tau_{kj}^{\theta} (\hat{w}_k \hat{P}_k)^{1-\eta} - \theta} \hat{w}_j \hat{Y}_j^{org} \bar{D}_j^{ctf}.
\] (37)

After solving for \(\hat{w}, \hat{P}\) using the procedure, changes to other macroeconomic variables including output \(\hat{Y}\) can be derived through their covariances with \(\hat{w}, \hat{P}\) from the equilibrium conditions of the model. The hat algebra method is easy to implement here and it only requires calibrating countries’ initial output \(\bar{Y}_{org}^i\) and bilateral trade shares \(\bar{\pi}_{ij}^{org}\) in the original steady state, which can be regarded as their long-term average values directly observable in the data.

This paper departs from the trade literature in the characterization of \(\bar{D}^{ctf}\), which is determined by equations 29 and 30. To solve the portfolio choice problem, I apply the hat algebra method around original and counterfactual steady states respectively to determine ‘intra-regime changes’ in equation 32. For intra-regime analysis, the initial levels of output and bilateral trade shares using which hat algebra is performed are set as their steady-state values in each regime \(\bar{Y}_s^i, \bar{\pi}_{ij}^s\), \(s \in \{org, ctf\}\). The original steady-state values are calibrated to the data. The counterfactual steady-state values will be predicted under inter-regime changes \(\hat{w}, \hat{P}\) imposed on the original steady state. In terms of notations, if the ratio of any variable \(A\) in response to simulated productivity shocks around a steady state in either original or counterfactual regime is denoted as

\[
\hat{A}_s^t = \frac{A_s^t}{\bar{A}_s^t}, \quad \text{where} \quad s \in \{org, ctf\},
\] (38)

For any country-specific variable \(B_{i,t}^s\), its cross-country ratio \(B_{i,j,t}^s\) defined in equation 15 has its percentage deviation approximated as

\[
\tilde{B}_{i,j,t}^s = \tilde{B}_{i,t}^s - \tilde{B}_{j,t}^s = \ln\left(\frac{\hat{B}_{i,t}^s}{\hat{B}_{j,t}^s}\right) = \ln\left(\hat{B}_{i,j,t}^s\right).
\] (39)

Given the notations, the variables that appear in the portfolio determination equations (29-30) can be converted from the tilde to the hat form. Since the variables in the portfolio equations are all expected to be realized at \(t+1\), I omit the time subscript for brevity in the following analysis. These variables in the hat form can be characterized as their responses to the vector of countries’ simulated productivity shocks

\[
\hat{T}' = [\hat{T}_1, \hat{T}_2, ..., \hat{T}_l] = \left[\frac{T_1}{T_1}, \frac{T_2}{T_2}, ..., \frac{T_l}{T_l}\right], \quad \forall i \in [1, I].
\] (40)
Elements in this vector \( \hat{T}_i \) will drive the dynamics of variables around the steady state in each regime \( s \in \{ \text{org, ctf} \} \) including wage and price:

\[
\tilde{P}_i^{1-\theta,s} = \sum_{j=1}^{I} \bar{\pi}^{s}_{ij} \hat{T}_i (\bar{\tilde{w}}^{\eta,s}_j \tilde{P}^{1-\eta,s}_j)^{-\theta},
\]

\[
\tilde{w}^{s}_i \bar{Y}_i^{s} = \sum_{j=1}^{I} \bar{\pi}^{s}_{ij} \hat{T}_i (\bar{\tilde{w}}^{\eta,s}_j \tilde{P}^{1-\eta,s}_j)^{-\theta} \tilde{Y}_j^{s}.
\]

It is worth noting that equations 41 and 42 for intra-regime analysis are counterparts to equations 36 and 37 for inter-regime analysis. Within either regime, tariffs and equilibrium portfolios are constant and therefore do not influence how variables behave around the steady state. Instead, the main driver for the dynamics of variables are productivity shocks \( \hat{T} \), under which \( \tilde{w}^s, \tilde{P}^s \) are solved with the same iterative procedure as for inter-regime analysis. I then express the changes of the second-moment variables in portfolio determination equations (listed in 31) as functions of \( \tilde{w}^s, \tilde{P}^s \) to derive asset changes across original and counterfactual regimes (\( \Delta D \)). Asset positions in the financial channel, by affecting countries’ expenditure in the trade channel (equation 7), will influence the real variables. Hence, I update inter-regime dynamics \( \hat{w}, \hat{P} \) with \( \Delta D \) using equations 36 and 37 until convergence is reached. The general equilibrium of this framework, where trade and financial channels interact with each other, can be characterized by the solution to a joint fixed-point problem of \( (\hat{w}, \hat{P}, \Delta D) \).

Dekle et al. (2007) follow the theorems of Alvarez and Lucas (2007) to establish the existence and uniqueness of the model solution to a fixed-point problem of \( (\hat{w}, \hat{P}) \) given counterfactual asset positions. Many properties of their numerical solutions are maintained under the assumptions specified in this model, for example there exists a unique price vector \( \tilde{P}^s \) within a regime given the corresponding wage vector \( \tilde{w}^s \). After solving the portfolio choice problem by evaluating the second-moment variables as functions of \( \tilde{w}^s \) in each regime \( s \in \{ \text{org, ctf} \} \), the resulting counterfactual portfolio \( D^{\text{ctf}} \) is then used.

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13 Alvarez and Lucas (2007) show that under the assumptions that \( \eta < 1, 1 + \theta (1 - \epsilon) > 0, \tau_{ij} \geq 1 \), a unique solution to counterfactual \( w \) exists to ensure zero excess demand (denoted as \( Z_i(w) \) in equation 43 for the level of wage) in the commodity market. They prove these theorems by showing that \( Z_i(w) \) is continuous, homogenous of degree zero, has the gross substitute property \( \frac{\partial Z_i(w)}{\partial w_i} > 0 \), satisfies Walras’s Law \( (\sum_i w_i Z_i(w) = 0) \), faces a lower but not upper bound \( Z_i(w) \rightarrow -\infty \), \( Z_i(w \rightarrow w^{org}) \rightarrow \infty \).
to characterize an excess demand system across regimes

$$Z_i(\hat{w}_i) = \frac{1}{\hat{w}} [\hat{w}_i \hat{Y}_i^{org} - \sum_{j=1}^{I} \frac{\hat{\pi}_{ij}^{org} \hat{r}_{ij}^{\eta} (\hat{w}_i \hat{P}_i^{1-\eta})^{-\theta}}{\sum_{k=1}^{I} \hat{\pi}_{kj}^{org} \hat{r}_{kj}^{\eta} (\hat{w}_k \hat{P}_k^{1-\eta})^{-\theta}} \hat{w}_j \hat{Y}_j^{org} \hat{D}^{ctf}_j], \quad (43)$$

$\hat{D}^{ctf}$ appears as a finite multiplier and hence does not change most properties described in footnote 13 of $\hat{w}$ computed as the solution to equation 43. For example, under the world resource constraint

$$\sum_{i=1}^{I} \hat{w}_i \hat{Y}_i^{org} = \sum_{i=1}^{I} \hat{w}_i \hat{Y}_i^{org} \hat{D}^{ctf}_i, \quad (44)$$

Walras’s Law is satisfied:

$$\sum_{i=1}^{I} \hat{w}_i Z_i(\hat{w}) = \sum_{i=1}^{I} (\hat{w}_i \hat{Y}_i^{org} - \sum_{j=1}^{I} \hat{\pi}_{ij}^{ctf} \hat{w}_j \hat{Y}_j^{org} \hat{D}^{ctf}_j) = 0, \quad (45)$$

which is necessary to establish the existence of the solution to inter-regime changes. The solution will characterize the counterfactual outcome $(\hat{w}, \hat{P}, \Delta \hat{D})$ which encompasses both the real and financial sides of the economy in policy experiments.

### 2.3 Algorithm

The computation algorithm used to solve the model is outlined as follows.

**Step 1. Collect data to calibrate the original steady state of the economy**

Obtain the data of country-level GDP and NFA, and of bilateral trade shares and bilateral portfolio weights (see Appendix B for data sources). The mean values over the sample period are used as the steady-state values of these variables in the original regime.

**Step 2. Form initial guesses about inter-regime changes**

Start with the guess that original and counterfactual regimes have the same steady-state values for wage, price, and asset positions

$$\hat{w}_0 = \hat{P}_0 = ones(I, 1), \quad \Delta \hat{D}^0 = zeros(I, 1), \quad (46)$$
Step 3. Characterize intra-regime changes

Simulate productivity shocks (see Appendix B for its calibration), solve for the responses of $\hat{w}^s, \hat{P}^s$ around original and counterfactual steady states respectively ($s \in \{\text{org, ctf}\}$) to the shocks. The original steady state is from step 1, and the counterfactual steady state including $\tilde{Y}^{ctf}, \tilde{\pi}^{ctf}$ is predicted from inter-regime changes ($\hat{w}$ and the corresponding $\hat{P}$ that satisfies 36) imposed on the original steady state. Compute $\hat{w}^s, \hat{P}^s$ in each regime with a recursive procedure based on 41 and 42 until convergence. Calculate the product of other variables including $\tilde{Y}^s, \tilde{R}^s$ as functions of the solved $\hat{w}^s, \hat{P}^s$ (see appendix A for their relations) in response to the simulated productivity shocks and take the mean values across simulations to get the second-moment variables.

Step 4. Solve the portfolio choice problem

Use the second-moment variables from step 3 in the original and counterfactual portfolio determination equations (29 and 30) to yield

$$E_t[(\gamma(1-\beta)\tilde{Y}\tilde{R}^s + (1-\gamma+\beta\gamma)\tilde{P}\tilde{R}^s + \gamma(1-\beta)\tilde{\alpha}^s\tilde{R}\tilde{R}^s] = \frac{1}{2}F^s, s \in \{\text{org, ctf}\}. \quad (47)$$

where $F^s$ and $\tilde{\alpha}^s$ stand for the vectors of all the countries’ financial frictions and asset holdings (defined by 24 and 28):

$$F^s = [F^s_{1I}, F^s_{2I}, ..., F^s_{I-I}], \quad \tilde{\alpha}^s = [\tilde{\alpha}^s_1, \tilde{\alpha}^s_2, ..., \tilde{\alpha}^s_{I-I}]. \quad (48)$$

$\tilde{Y}\tilde{R}^s, \tilde{P}\tilde{R}^s,$ and $\tilde{R}\tilde{R}^s$ represent the second-moment variables listed in 31 for brevity. Take the difference of equation 47 between the two regimes to determine the shift of bilateral asset holdings driven by second-moment variables including potential changes to financial frictions. Add up bilateral holdings to yield country-level aggregate asset positions, which will update $\Delta \tilde{D}^0$ to $\Delta \tilde{D}^1$.

Step 5. Update Inter-regime changes given the solved portfolio

Update inter-regime changes of $\hat{w}$ using the portfolio $\Delta \tilde{D}$ from step 4. This involves

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The purpose of this step is to calculate second-moment variables (listed in 31) that appear in portfolio determination equations (29 and 30). To do the calculation, we need the percentage deviation of variables from a steady state under the shocks of the economy.
employing an iterative computation procedure similar to that described by Alvarez and Lucas (2007) and Dekle et al. (2007) with function \( M \), which denotes the mapping of \( \hat{\mathbf{w}}^0 \) to \( \hat{\mathbf{w}}^1 \) with a constant \( \nu \in (0, 1) \) and excess demand \( \mathbf{Z}_i(\hat{\mathbf{w}}) \) defined in equation 43:

\[
\hat{\mathbf{w}}^1 = M(\hat{\mathbf{w}}^0) = \hat{\mathbf{w}}^0 (1 + \nu \frac{\mathbf{Z}_i(\hat{\mathbf{w}}^0)}{\bar{Y}_{\text{org}}}).
\] (49)

Note that the mapping is bounded by one under Walras’s Law (equation 45)

\[
\sum_{i=1}^{I} M(\hat{\mathbf{w}}_i^0) \bar{Y}_i^\text{org} = \sum_{i=1}^{I} \hat{\mathbf{w}}_i^0 (1 + \nu \frac{\mathbf{Z}_i(\hat{\mathbf{w}}_i^0)}{\bar{Y}_i^\text{org}}) \bar{Y}_i^\text{org} = \sum_{i=1}^{I} \hat{\mathbf{w}}_i^0 \bar{Y}_i^\text{org} + \nu \sum_{i=1}^{I} \hat{\mathbf{w}}_i^0 \mathbf{Z}_i(\hat{\mathbf{w}}_i^0) = 1, \quad (50)
\]

and the normalization condition that treats the world output as a numeraire

\[
\sum_{i=1}^{I} \hat{\mathbf{w}} \bar{Y}_i^\text{org} = 1. \quad (51)
\]

**Step 6. Repeat steps 3-5 until convergence**

Use the updated \( \hat{\mathbf{w}}^1 \) from step 5 and \( \Delta \bar{D}^1 \) from step 4 as new guesses, and repeat the procedures from step 3 to 5 for both inter-regime and intra-regime analyses to reach new updated \( \hat{\mathbf{w}}^2, \hat{\mathbf{P}}^2 \) and \( \Delta \bar{D}^2 \). This continues until the difference between the \( k^{th} \) and the \( k+1^{th} \) iteration \( | \hat{\mathbf{w}}^{k+1} - \hat{\mathbf{w}}^k |, | \hat{\mathbf{P}}^{k+1} - \hat{\mathbf{P}}^k |, | \Delta \bar{D}^{k+1} - \Delta \bar{D}^k | \) is sufficiently small, which solves the joint fixed-point problem of \( (\hat{\mathbf{w}}, \hat{\mathbf{P}}, \Delta \bar{D}) \) necessary to characterize counterfactual outcomes under alternative tariffs or financial frictions.

### 3 Policy Experiments

This section conducts policy experiments to 1) explain the mechanism of how trade and financial frictions influence cross-country economic linkages, and 2) highlight the departure of this unified trade-finance model from standard international trade and macro frameworks. I calibrate the model to a world economy that consists of 43 countries (listed in table B.1) plus the rest of the world (ROW) to perform the policy experiments.
3.1 Calibration

To calibrate the real side of the economy, I only need countries’ GDP data from the Penn World Table (PWT) and bilateral trade shares from the Direction of Trade Statistics when using the hat algebra technique to predict counterfactuals. Analogously on the financial side, I obtain countries’ net foreign asset positions (NFA) from the World Bank and bilateral portfolio weights from Factset/Lionshare (see Appendix B for details on data and calibration). The time-averaged values over the sample period 2001-2021 are used as the steady-state values of these variables in the original regime.

The risk characteristics of the economy are reflected as productivity fluctuations. Therefore, I estimate countries’ dynamic productivity consistent with the Eaton and Kortum (2002) model following Levchenko and Zhang (2014)’s approach and compute its corresponding mean value and covariance matrix (see Appendix B). Productivity shocks necessary for intra-regime analyses can either be simulated with a joint normal distribution featuring the estimates or directly with the bootstrap method.\(^{15}\) Other parametric assumptions include the annual discount factor \(\beta = .9\), coefficient of relative risk aversion \(\gamma = 2\), share of intermediate input in production \(\eta = .312\) following Dekle et al. (2007), share of labor input \(1 - \mu\) as country-specific labor income share from the PWT, and trade elasticity \(\theta = 4\) based on Simonovska and Waugh (2014).

3.2 Counterfactual Financial Frictions

I start the policy experiments by examining how portfolio choice reacts to financial frictions. In particular, I impose a uniform increase of bilateral financial frictions among all the country pairs by

\[
\tilde{F} = \frac{F^{\text{ctf}} - F^{\text{org}}}{F^{\text{org}}}. \tag{52}
\]

Note the changes in bilateral frictions are in relative terms to the frictions faced by the numeriare country \(I\) which is assumed to be ROW, as the element in the \(i^{th}\) row \(j^{th}\) column of \(F^s\) in regime \(s \in \{\text{org, ctf}\}\) (jointly defined by equations 22, 24, and 48) is

\[
F^s(i, j) = (f_{II} - f_{ij}) - (f_{II} - f_{iI}). \tag{53}
\]

\(^{15}\)Bootstrap is more feasible when the estimated covariance matrix is not positive semi-definite, due to the collinearity problem given the large-scale data, to simulate random samples from the specified multivariate normal distribution.
Everything else equal, an increase in bilateral friction $f_{ij}$, which stands for country $i$’s friction when holding $j$’s asset, will generate a negative $\tilde{F}(i, j)$. The counterfactual exercise assumes $\tilde{F}(i, j) = -0.2$ for $\forall i \neq j \in [1, I]$, which represents a universal 20% increase of bilateral financial frictions relative to the numeraire country.

It is useful to discuss economic mechanism before showing numerical results to understand the drivers for financial allocation. To examine asset changes under counterfactual frictions, I simplify the portfolio determination equation 47 in either regime as

$$E_t[(\gamma(1 - \beta)\tilde{R}\tilde{R}'s + (1 - \gamma + \beta\gamma)\tilde{P}\tilde{R}'s) = \frac{1}{2} F^s, s \in \{org, ctf\}]^{16} (54)$$

under the model assumption that financial income flows are proportional to output based on equation 8 such that

$$\tilde{Y}\tilde{R} = \tilde{Y}\tilde{Y}' = \tilde{R}\tilde{R}' . (55)$$

If inter-regime changes of variables are marked with a $\Delta$, the difference of portfolio equations across regimes is given by

$$E_t[(\gamma(1 - \beta)\Delta\tilde{R}\tilde{R}' (1 + \Delta\tilde{X}) + (1 - \gamma + \beta\gamma)\Delta\tilde{P}\tilde{R}' ) = \frac{1}{2} \tilde{F}F^{org} . (56)$$

From this equation, an increase in financial frictions will generate a decrease in the corresponding asset holdings $\tilde{\alpha}$. Nonetheless, the magnitude of the decrease also depends on the second-moment variables, which simultaneously shift across policy regimes under the influence of the frictions to restore equation 56. Specifically, asset holdings will drop less if $\Delta\tilde{R}\tilde{R}' < 0$. To understand the intuition, the term $\tilde{R}\tilde{R}'$ is the asset covariance matrix and therefore captures the comovement across countries’ financial returns. When its elements decrease, financial assets provide more risk-sharing benefits since their returns covary less across countries. Therefore, by constructing diversified portfolios of different countries’ assets, households have more stable financial income less subject to country-specific risks for consumption smoothing. This creates greater incentives for households to hold financial assets despite higher financial frictions. Meanwhile, asset holdings also decline less when $\Delta\tilde{P}\tilde{R}' > 0$ and if households are sufficiently risk averse.$^{17}$ The term

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$^{16}$Note that all the variables of second order including $\tilde{R}\tilde{R}'$, $\tilde{P}\tilde{R}'$, and $F^s$ will be $(I - 1) \times (I - 1)$ matrices. $\tilde{R}\tilde{R}'$’s and $\tilde{P}\tilde{R}'$’s element in the $i^{th}$ row $j^{th}$ column captures the comovement of country $i$’s own return or price with $j$’s asset returns under all the productivity shocks in the economy, which will determine $i$’s holding of $j$’s assets affected by the financial friction also in the $i^{th}$ row $j^{th}$ column of $F^s$.

$^{17}$Risk aversion decides when domestic goods become more expensive, whether households want to generate more income to finance current expenditure or to postpone consumption for intertemporal
Δ\tilde{P}\tilde{R}' captures the comovement of countries' inflation with asset returns. When it increases, financial assets provide greater risk-hedging benefits, since financial income is higher when domestic goods are more expensive which is helpful to sustain households' purchasing power under price fluctuations. This mechanism is also referred to as hedging against real exchange rate risk which is an important explanation for asset home bias (see Coeurdacier and Rey (2013)). Nonetheless, this risk-hedging mechanism plays a significantly more essential role in determining preference between home and foreign assets in a two-country framework than in this multi-country model where households choose assets from many countries. The correlations between foreign assets and domestic inflation are low and do not vary much across foreign countries in this setting. Therefore, I will focus more on the risk-sharing mechanism shaped by Δ\tilde{R}\tilde{R}' in the following analysis.

Figure 1 presents the changes in portfolios and in cross-country asset covariances under counterfactual financial frictions. (a) plots the US bilateral holdings of other countries' assets (\hat{\alpha}_{ij} = \frac{\alpha_{ij}}{\hat{\alpha}_{ij}}) and asset covariances with those countries. When faced with higher financial frictions with foreign countries, the US holding of domestic assets more than doubles: \hat{\alpha}_{US,US} > 2, but the increase is not as large as its holding of Brazil’s and New Zealand’s assets with which the return covariances sharply decrease. In contrast, the US cuts holdings of assets from countries with which covariances increase significantly including Kuwait and Bahrain. The overall negative correlation between \hat{\alpha} and Δ\tilde{R}\tilde{R}' verifies the risk sharing mechanism discussed earlier. Besides the composition of portfolio at the bilateral level, I also examine the size of portfolio which adds up bilateral positions to the country level. Figure (b) shows countries’ aggregate asset holdings and their median asset covariance across partners. Overall there is a negative correlation between the changes in the two variables: Countries with a higher median asset covariance increase with others are expected to reduce financial holdings by a greater magnitude under counterfactual financial frictions. On the contrary, countries which yield more risk-sharing benefits, reflected by a lower Δ\tilde{R}\tilde{R}', are less likely to curtail asset positions in response to the increased financial frictions.

This analysis of how financial investment is affected by cross-country correlations echoes related works in international finance including Coeurdacier and Guibaud (2011) and Bergin and Pyun (2016). This paper contributes to this literature by proposing a multi-country structural framework useful to quantify the impacts of frictions on both portfolio choice and underlying covariance structure.

substitution. If they are sufficiently risk averse (\gamma > \frac{1}{1-\beta} here), the former dominates the latter effect, and vice versa.
Figure 1: Portfolio and cross-country covariance changes under financial frictions

This figure plots the counterfactual changes in portfolio and in cross-country covariance matrix $\Delta \tilde{R}'$ under changes in financial frictions $\tilde{F} = -0.2$, $\forall i \neq j \in [1, I]$. (a) plots the US bilateral holdings of other countries' assets ($\tilde{\alpha} = \tilde{\alpha}_{ctf}$) and its asset return covariances with others. (b) plots all the countries' aggregate asset positions $\Delta \tilde{D}$ and their median covariances with others.

Besides financial variables, real variables shift across policy regimes under counterfactual financial frictions, because inter-regime changes are characterized as the simultaneous changes to the real and financial sides of the economy in the general equilibrium. Figure 2 plots cross-country wage ($\tilde{\bar{w}}$) and price ($\tilde{\bar{P}}$) changes under counterfactual financial frictions. Affected by the frictions, most countries in the diagram experience lower wage and price compared to the original regime ($\tilde{\bar{w}}, \tilde{\bar{P}} < 1$). Between the two variables, many countries’ loss in wage is larger than that in price ($\tilde{\bar{w}} < \tilde{\bar{P}}$) as they lie above the 45 degree line. A few exceptions include Canada and US which witness wage increases. Several potential factors can contribute to this result, including the fact that these two countries’ financial holdings which can be used to finance expenditure rise remarkably, which drives up domestic demand for goods that consequently raises labor compensation. This magnitude of the changes is jointly shaped by all their bilateral linkages with other countries through trade and financial channels. Except for these outliers, most countries are predicted to have lower wage under higher financial frictions based on the model.

3.3 Counterfactual Tariffs

To further examine how trade and financial channels interact in the general equilibrium, this section conducts a policy experiment where frictions come from the real side of the economy. Specifically, I assume bilateral tariffs among all the country pairs in the
Figure 2: Wage and price changes under financial frictions

This figure plots the counterfactual changes in wage $\hat{w}$ and price $\hat{P}$ under changes in financial frictions $F = -0.2, \forall i \neq j \in [1, I]$.

Figure 3 presents the comparative statics under varying tariffs for a group of selected countries across continents. Under higher tariffs, wage across the selected countries declines in general but at different rates. Not surprisingly, China as a major exporter has wage losses most sensitive to tariff changes. In terms of finance, all the selected countries except Kuwait document an increase in asset positions $\Delta D > 0$ by around 5% to 25% as shares of GDP under tariff changes, which suggests that these economies borrow more to finance expenditure. Meanwhile, asset positions are not as responsive as wage to tariffs reflected in the smaller slope of the curves in (b), especially for Australia and the US. To diagnose the pattern, I compare countries’ changes in asset positions ($\Delta D$) and in their median asset covariance with others ($\Delta RR'$) in table 1. From the table, $\Delta RR'$ of Australia and the US is relatively stable which contributes to the flatness of their financial curves under tariff changes. Moreover, Kuwait is the only country that witnesses declines in asset holdings, which can be attributed to the large increase in the value of its $\Delta RR'$. Based on the earlier discussion, the increase infers worse risk-sharing benefits brought
forth by financial assets, which induces the country to shrink holdings.

Next I combine trade and financial channels to conduct welfare analysis. The trade literature including Dekle et al. (2007) measures welfare, with both cost and size of expenditure considered, as the wage-to-price ratio multiplied by the asset position:

$$\widehat{W}_i = \frac{\widehat{w}_i}{\widehat{P}_i} \frac{\Delta D_{i}^{c}}{\Delta D_{org}^{c}}.$$  (58)

The two fractions in this definition of welfare stand for the changes on the real and financial side of the economy respectively. Using this decomposition, I compare the counterfactual welfare derived under fixed asset positions ($\Delta D = 0$) and under adjustable positions determined by households’ optimal investment decisions under tariff changes. The comparison quantifies the departure of this new approach, which endogenizes cross-country financial flows, from the trade literature that treats countries’ asset positions as exogenous and fixed.

Figure 4a compares the counterfactual wage-to-price ratio under fixed and adjustable asset positions. Most countries’ ratio is lower in either case ($\frac{\widehat{w}}{\widehat{P}} < 1$) given a tariff change $\hat{\tau} = 1.2$ relative to the original situation without tariffs. This is because higher tariffs prohibit cross-country commodity flows and raise the price paid by households more than their wage. Between the two cases, the wage-to-price ratio decreases by a greater magnitude under adjustable than under fixed asset positions as most countries lie above
This figure plots the counterfactual changes to wage-to-price ratio $\bar{\bar{w}} / \bar{P}$ in (a), counterfactual equilibrium asset positions as shares of output $\bar{D}^{ctf}$ in (b), and counterfactual welfare changes $\bar{W}$ in (c) when bilateral tariffs in the world economy uniformly increase by 20%: $\bar{\tau} = 1.2$. Variables on the horizontal axis represent the case where asset positions are adjustable in response to tariffs and the variables on the vertical axis represent the case where asset positions are fixed.
Table 1: Asset covariance and position changes under different tariffs

<table>
<thead>
<tr>
<th>Country</th>
<th>Variable</th>
<th>$\hat{\tau} = 1$</th>
<th>$\hat{\tau} = 2$</th>
<th>$\hat{\tau} = 3$</th>
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<tr>
<td>Australia</td>
<td>$\Delta \tilde{R}'$</td>
<td>-0.211</td>
<td>-0.199</td>
<td>-0.189</td>
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<tr>
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<td>$\Delta \tilde{D}$</td>
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<td>0.072</td>
<td>0.072</td>
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<td>0.174</td>
<td>0.185</td>
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<tr>
<td></td>
<td>$\Delta \tilde{D}$</td>
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<td>0.230</td>
<td>0.199</td>
</tr>
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<td>$\Delta R'$</td>
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<td>2.006</td>
<td>2.245</td>
</tr>
<tr>
<td></td>
<td>$\Delta \tilde{D}$</td>
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</tr>
</tbody>
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This table reports the changes in the median covariance of asset returns $\Delta \tilde{R}'$ across pairs a country makes and in its asset position $\Delta \tilde{D}$ under different tariff changes $\hat{\tau}$ relative to the original scenario.

The 45 degree line in the diagram. This result suggests that households witness greater deteriorations to the purchasing power of their labor income when they can adjust their financial holdings in response to tariffs. Nonetheless, financial income partially offset the impact of labor income loss on households’ welfare, as figure 4b shows that most countries increase their asset holdings to raise expected lifetime utility. This happens since assets provide more risk-sharing benefits (reflected as $\Delta \tilde{R}' < 0$) when the trade channel, which facilitates international output synchronization that generates a higher $\tilde{R}'$, faces greater barriers. Consequently, most countries increase asset holdings, in which process they resort to finance for international risk sharing when trade relations deteriorate.

Figure 4c illustrates the welfare change which combines the wage-to-price ratio in figure 4a and the asset position in figure 4b. If the original welfare without tariff increase is normalized to 1, the median counterfactual welfare across countries is 0.782 under fixed and 0.818 under adjustable asset positions. Except for a few countries such as Finland and Bahrain, most countries have higher welfare when financial holdings can be adjusted.

---

18 It is worth noting that figures 4a and 4b should be examined simultaneously because they are both derived from the model solution to a joint fixed-point problem of $(\hat{\tilde{w}}, \hat{\tilde{P}}, \Delta \tilde{D})$ under the normalization condition that treats the world output as a numeraire $\sum_{i=1}^{I} \tilde{Y}_i = 1$ and the world resource constraint $\sum_{i=1}^{I} \tilde{w} \tilde{Y}_i \tilde{Y}_i = \sum_{i=1}^{I} \tilde{w} \tilde{Y}_i \tilde{Y}_i \tilde{D}_i$. Under these two conditions, countries’ increased borrowing is sustainable since their output and wage income decline concurrently under tariffs. Countries’ new asset positions reflect households’ optimal utility maximization decisions given the tariff changes.
(figure 4c). Hence, the welfare loss of these countries would be overestimated under
the assumption that their asset positions were fixed given tariff changes. For example,
Qatar’s welfare loss would be doubled if potential adjustments to its financial holding
were not taken into account.

The discrepancy between the two cases occurs because a standard trade model ex-
cludes finance, which is a major means for risk-averse agents to achieve consumption
smoothing for utility maximization. When the world economy faces higher tariffs which
hinder output synchronization, agents endogenously increase asset holdings under in-
tertemporal decisions to take advantage of the risk-sharing benefits given lower cross-
country covariances in the financial market. Ignoring this margin of adjustment may
miscalculate welfare. Therefore, this comparison between fixed and flexible asset posi-
tions underscores the importance of incorporating a financial channel in a quantitative
trade model. Meanwhile, the comparison also delivers policy implications for capital ac-
count openness which is a classic question of interest in international macroeconomics.19

To further capture the departure of this framework from a conventional international
macro model with a small number of countries, figure 5 compares the counterfactual
welfare and that from a two-economy scenario. When calibrating the two-economy model,
I treat each country as the domestic economy and the sum of all the other countries from
this country’s perspective as the foreign economy. Following this rule, I calculate the two-
by-two matrices of financial and trade shares, and re-estimate the productivity of domestic
and foreign economies based on trade flows for each country. As is shown in figure 5a, the
real wage (\(\hat{\bar{w}}_i\)) of Belgium, Netherlands, and Ireland is smaller in a multi-economy scenario.
Therefore, modeling these European countries, with their heavy reliance on numerous
economic partners, as small open economies interacting with one foreign country (the
world) may under-estimate the negative impact of tariffs on them. Meanwhile, figure 5b
suggests that most countries will increase their asset holdings by a greater amount in the
multi-economy case, largely thanks to the diversification benefits offered by all the cross-
country covariances which are not fully captured in the two-economy setup. As a result,
counterfactual welfare in figure 5c is also higher for the majority of the countries in the
multi-economy scenario. This comparison between multi- and two-economy frameworks
highlights the importance of considering multilateral linkages across all the countries in
developing open economy macro models. The solution technique proposed in this paper
makes it easier to conduct counterfactual analyses of a large-scale world economy.

19See, for example, Kose et al. (2009) for a survey on the benefits and costs of financial globalization
for developing countries.
Figure 5: Comparison of counterfactual welfare in two-economy and multi-economy cases

This figure plots the counterfactual changes to wage-to-price ratio $\hat{\bar{w}}$ in (a), counterfactual equilibrium asset positions as shares of output $\hat{D}^{ctf}$ in (b), and counterfactual welfare changes $\hat{W}$ in (c) when bilateral tariffs in the world economy uniformly increase by 20%: $\tau = 1.2$. Variables on the horizontal axis represent the case where there are 43 economies and the variables on the vertical axis represent the case where there are two economies with a country itself as the domestic economy and the sum of all the other countries as the foreign economy.
4 China-US Trade War

This section examines the recent China-US economic conflict as a real-world application of the model. In 2018, the Trump administration raised tariffs and other trade barriers on China, which took retaliatory actions in response. Besides reciprocal tariff increases, selling off US assets was considered a possible means for China to strike back.\footnote{Media coverage on this possibility can be found in New York Times, Forbes, Politico, among many others. Nearly all these media agree that although the possibility is not high, the consequences can be catastrophic for both countries once China weaponizes its portfolio ownership including over 1 trillion dollars US government debt.} I conduct analyses with both trade and financial measures based on the structural model to quantify the welfare implications of the trade war.

I obtain the bilateral tariff changes during the trade war from Li et al. (2020), who merge industry-level tariff changes from government announcements with sectoral trade data from the UN Comtrade, to calculate the trade-weighted cumulative tariff increases by March 2020. Based on this method, the trade-weighted average US tariff increase on Chinese exports is 18.6\%, and Chinese retaliatory tariff increase on US goods is 18.9\%. Therefore, I set bilateral tariff changes as $\hat{\tau}_{US,CN} = 1.189, \hat{\tau}_{CN,US} = 1.186$ respectively. In the financial channel, besides the situation where all the countries including China adjust portfolios based on risk-sharing incentives shaped by the new global trade pattern under the tariffs, I also consider the extreme scenario where China sells all its US asset holdings and reconstructs the portfolio among the remaining investment destinations.

Figure 6 plots the model-predicted real and financial variables of the global economy during the trade war. Although the tariff changes only occur between China and the US, all the countries besides the two major economies are affected. \footnote{For example, Caliendo and Parro (2021) estimate with a multi-sector multi-region quantitative spatial model that U.S. aggregate real wages decline by 0.16 percent due to the trade war. Li et al. (2020) build a computational general equilibrium (CGE) model to predict that China’s welfare falls by 1.7\%. Other countries have disparate but small changes to real wage.} 6a shows that most countries witness lower wage and price due to the tariff increases as $\hat{P}, \hat{w} < 1$. Furthermore, the majority of the countries suffer loss in real wage as $\hat{w} < \hat{P}$. The median value of $\frac{\hat{w}}{\hat{P}}$ in the sample is 0.8, representing the mean value of Spain and Norway. The magnitude of wage changes is overall larger than what is predicted from the existing literature for two major reasons. First, this model assumes a single tradable sector, while many trade models consider both nontradable and tradable sectors with global input-output linkages. Cross-country cross-sector substitution in a sophisticated quantitative trade framework can potentially reduce the impacts of industry-specific tariffs and leave wage and price
less affected. Second and more importantly, this paper considers the endogenous changes to countries’ asset positions while most existing literature ignores this margin of adjustment. Figure 6b shows that most countries increase their asset holdings in response to the tariff changes if cross-country financial frictions remain the same. The earlier explanation for figure 4b also applies here: cross-country asset covariances decrease when tariffs increase which impair output synchronization. Hence, countries have stronger incentives to hold each others’ assets for international risk sharing. This financial allocation requires that wages be adjusted to a lower level to satisfy the world resource constraint, which magnifies the adjustment of real variables compared to in a standard trade model.

I proceed to discuss the possibility that China gives up its holding of US assets as means of financial retaliation. To conduct this counterfactual analysis, I set $\alpha_{CN,US} = 0$ and solve for China’s portfolio choice among the remaining countries’ assets under their covariance structure. Figure 7 plots the difference in the changes of China’s asset holdings between the cases with and without financial retaliation. If China had to reconstruct its portfolio, the model predicts that it would replace US assets with either domestic assets or the assets of oil exporters including UAE, Bahrain, and Kuwait, while keeping most of the other asset positions slightly lower. Compared to the case with no financial retaliation, China’s aggregate asset holding $\tilde{D}^{ctf}$ would drop more by 0.16% as shares of GDP and its wage-to-price ratio would drop by 1.14%. Therefore, based on the welfare definition from equation 58, China’s welfare loss during the trade war would be exacerbated by 1.3%
This figure plots the difference in the changes to China’s holdings of assets between the case with financial retaliation (denoted as $ctf_1$ below) where $\alpha_{CN,US} = 0$ and the case without relation ($ctf_2$), both relative to the situation where there is no trade war ($org$). The values presented in the figure are calculated as $\Delta \alpha_{CN,i} = \frac{\alpha_{ctf_1} \alpha_{org} - \alpha_{ctf_2} \alpha_{org}}{\alpha_{org}}$, $\forall i \in [1, I]$ if the country retaliated in the financial channel. Nevertheless, this estimate does not consider complications beyond the scope of this model, including the influence of the asset sale on Chinese exchange rate policy or on the US monetary policy (in particular interest rate). Understanding such policy-relevant questions requires incorporating additional modeling ingredients and techniques from open economy macro frameworks, which will be interesting topics to explore for future research.

5 Conclusion

This paper combines the portfolio choice solution method and hat algebra technique to solve a unified multi-country model where finance and trade influence each other. This new approach can readily be applied to a wide range of topics in both international macroeconomics and trade literatures, meanwhile it has many potentials for extensions. I hereby discuss two directions for future work.

First, this paper focuses on comparative statics across steady states under specific policy regimes, without tracing the dynamic path of the world economy between steady states. Although the portfolio choice problem considers agents’ intertemporal investment
decisions, the derived equilibrium portfolio is static in nature. If future research questions involve time-series patterns of economic activities, solving dynamic portfolios requires extending the current method to higher-order approximations of the model. Devereux and Sutherland (2010) show that the first-order dynamics of the equilibrium portfolio is obtained by combining a third-order approximation of the portfolio determination equations with a second-order approximation to the rest of the model. On the real side of the economy, either ‘dynamic exact hat algebra’ techniques from the quantitative trade literature (Caliendo et al. (2019) and Kleinman et al. (2021) among others) or perturbation methods from the extensive DSGE literature (see Fernández-Villaverde et al. (2016) for a comprehensive survey) can be applied, depending on the specific context of interest. Such dynamic analyses characterize the world economy’s pattern and speed of convergence towards a steady state, which is important in quantifying both persistent and transitory economic outcomes when equilibrium is being restored.

Second, this paper uses the local perturbation method to derive portfolio choice around a deterministic steady state. The method offers a powerful yet tractable toolkit widely applicable to DSGE models. Furthermore, the derived solution is extremely close to the exact around the point of approximation, but is less accurate when there are large deviations from the steady state or when the problem exhibits strong non-linearity (see a detailed discussion in Coeurdacier and Rey (2013)). Therefore, if global solution methods (such as policy or value function iterations) for the portfolio choice problem with comparable applicability and tractability become available in the future, financial investment can be endogenously determined in more general economic environments. Important questions including those related to sovereign defaults and disaster risks can be answered with the development of such solution techniques.
References


Appendices

A Model Equations

In this section I describe the relationship among all the hat variables to set up the fixed-point problem for counterfactual exercises. Unless otherwise noted, a variable marked with a hat denotes the ratio of its value to the original steady state marked with a bar:

\[ \hat{A} = \frac{A}{\bar{A}}. \] (A.1)

I do not distinguish between inter- or intra-regime (\( \hat{A} \) or \( \hat{A}^s, s \in \{\text{org}, \text{ctf}\} \)) changes here because the equations below can characterize either case.

On the production side, factor prices are determined by firms’ profit maximization

\[ \frac{w_i L_i}{r_i K_i} = 1 - \frac{\mu}{\mu}. \] (A.2)

Therefore, the change in capital rental fee equals that in wage for this endowment economy

\[ \hat{r}_i = \hat{w}_i. \] (A.3)

These factor prices are reflected in a country’s income

\[ Y_i = w_i L_i + r_i K_i, \] (A.4)

which, with the definition of hat variables (equation A.1) and A.3, is re-written as

\[ Y_i = \hat{w}_i \bar{w}_i L_i + \hat{r}_i \bar{r}_i K_i = \hat{w}_i (1 - \mu) \bar{Y}_i + \hat{w}_i \mu \bar{Y}_i. \] (A.5)

Hence the change in income also equals that in wage

\[ \hat{Y}_i = \frac{Y_i}{\bar{Y}_i} = \hat{w}_i. \] (A.6)

Given these ingredients, I characterize the changes to wage (\( \hat{w}_i \)) and price (\( \hat{P}_i \)) which, with minor modifications, can be applied to the analysis both within (equations 36 and 37) and across (equations 41 and 42) regimes. With equations 4 and 5, price in country
\( \hat{w}_i \) is given by
\[
P_i^{-\theta} = \Gamma^{-\theta} \sum_{j=1}^{I} T_j \tau_{ji}^{-\theta} (w_i^{-\mu} r_j^{-\mu(1-\mu)} P_j^{-\eta})^{-\theta}, \quad (A.7)
\]

Divide this by \( P \)'s steady state value while imposing A.3 yields the change of price
\[
\hat{P}_i^{-\theta} = \sum_{j=1}^{I} \hat{\pi}_{ji} \hat{T}_j \tau_{ji}^{-\theta} \hat{w}_j^{-\mu} \hat{P}_j^{-\eta})^{-\theta}. \quad (A.8)
\]

Meanwhile, wage \( \hat{w}_j \) in the equation is derived from the goods market clearing condition. Plugging equation 4 in 6 and using \( X_i = Y_i \hat{D}_i \) gives
\[
Y_i = \sum_{j=1}^{I} T_i (\tau_{ij} (r_i^{-\mu} w_i^{-\mu}) P_i^{-\eta})^{-\theta} \hat{D}_j. \quad (A.9)
\]

Combining equations A.3, A.6, and A.8 yields its change
\[
\hat{\pi}_i \hat{Y}_i = \sum_{j=1}^{I} \sum_{k=1}^{I} \hat{\pi}_{ij} \hat{T}_i \tau_{ij}^{-\theta} \hat{w}_j^{-\mu} \hat{P}_j^{-\eta})^{-\theta} \hat{\pi}_{kj} \hat{T}_k \tau_{kj}^{-\theta} \hat{w}_k^{-\mu} \hat{P}_k^{-\eta} \hat{D}_j. \quad (A.10)
\]

In the financial channel, countries’ income determines their financial return defined by equations 8 and 9. Around the steady state of the economy, it is sufficient to solve for the equilibrium portfolio by evaluating a static wealth constraint where equity prices \( p^E \) drop out.\(^{22}\) Therefore, asset returns are determined by dividends as financial income which are proportional to real income. It follows immediately that the changes of these variables are the same:
\[
\hat{R}_i = \hat{Y}_i. \quad (A.11)
\]

After the joint \( (\hat{w}, \hat{P}, \Delta D) \) across regimes is solved using the stated algorithm, changes to welfare measured as the wage-to-price ratio adjusted for the size of expenditure is obtained from
\[
\hat{\mathcal{W}}_i = \frac{\hat{w}_i \Delta D_{t}^f}{\hat{P}_i \Delta D_{t}^{\mu g}}. \quad (A.12)
\]

The derivation of equations on the financial side is modified from the two-country

\(^{22}\text{Coeurdacier (2009) shows that the equilibrium portfolio in a static setup where return } R_{i,t} \text{ is determined by dividend } d_{i,t} \text{ is identical to that in a dynamic model for two reasons. First, the first-order dynamics of non-portfolio equations in the dynamic model remain the same as in the static model. Second, the present value of the dynamic budget constraint is satisfied up to a first-order if the static constraint holds.} \)
analysis by Devereux and Sutherland (2011). The first-order dynamics of the wealth constraints (equation 13) in this model is

\[
\tilde{W}_{i,t+1} = \frac{1}{\beta} \tilde{W}_{i,t} + \sum_{k=1}^{l-1} \tilde{\alpha}_{ik} (\tilde{R}_{k,t+1} - \tilde{R}_{I,t+1}) + \tilde{Y}_{t,t} - \tilde{P}_{t,t} - \tilde{C}_{i,t} + \mathcal{O}(\epsilon^2), \tag{A.13}
\]

\[
\tilde{W}_{I,t+1} = \frac{1}{\beta} \tilde{W}_{I,t} + \sum_{k=1}^{l-1} \tilde{\alpha}_{Ik} (\tilde{R}_{k,t+1} - \tilde{R}_{I,t+1}) + \tilde{Y}_{I,t} - \tilde{P}_{I,t} - \tilde{C}_{I,t} + \mathcal{O}(\epsilon^2), \tag{A.14}
\]

where wealth is normalized by a country’s consumption \(\tilde{W}_{i,t} = (W_{i,t} - \bar{W}_i)/\bar{C}_i\) and portfolio weights are scaled by the holder country’s output and discount factor:

\[
\tilde{\alpha}_{ik} = \frac{\alpha_{ik}}{\beta Y_i}, \tag{A.15}
\]

which constitute the country’s vector of asset holdings

\[
\tilde{\alpha}_i = [\tilde{\alpha}_{i1}, \tilde{\alpha}_{i2}, ..., \tilde{\alpha}_{il-1}]. \tag{A.16}
\]

Taking the difference between equations A.13 and A.14, iterating forward over the infinite time horizon, and imposing a transversality condition to drop \(\tilde{W}_{i,t+\infty}\) yields

\[
\sum_{s=0}^{\infty} \beta^s E_{t+1}(\tilde{C}_{i,I,t+1+s}) = \frac{1}{\beta} \tilde{W}_{i,I,t} + \tilde{Y}_{i,I,t+1} - \tilde{P}_{I,I,t+1} + \sum_{k=1}^{l-1} (\tilde{\alpha}_{ik} - \tilde{\alpha}_{Ik})(\tilde{R}_{k,t+1} - \tilde{R}_{I,t+1}) + \mathcal{O}(\epsilon^2), \tag{A.17}
\]

which gives the expected consumption differential between the two countries

\[
\tilde{C}_{i,I,t+1} = \frac{1 - \beta}{\beta} \tilde{W}_{i,I,t} + (1 - \beta) \tilde{Y}_{i,I,t+1} - (1 - \beta) \tilde{P}_{I,I,t+1} + (1 - \beta) \sum_{k=1}^{l-1} (\tilde{\alpha}_{ik} - \tilde{\alpha}_{Ik})(\tilde{R}_{k,t+1} - \tilde{R}_{I,t+1}) + \mathcal{O}(\epsilon^2). \tag{A.18}
\]

Plugging it in the second-order approximation of the Euler equation (23) yields an equation to determine the portfolio for time \(t+1:\)

\[
E_t[(\gamma(1 - \beta) \tilde{Y}_{i,I,t+1} + (1 - \gamma + \beta \gamma) \tilde{P}_{I,I,t+1} + \gamma(1 - \beta)(\tilde{\alpha}_i - \tilde{\alpha}_I) \tilde{R}_{x,t+1}) \tilde{R}_{x,t+1}] = \frac{1}{2} F_{t}, \tag{A.19}
\]

whose components can be evaluated with the hat algebra technique around the steady state of the economy.

37
B Data and Calibration

This section describes the data source and calibration strategy for both the real and financial sides of the economy. The sample of economies includes 43 countries (listed in table B.1) and the rest of the world (ROW). The time-averaged values of variables over the sample period from 2001-2021 will be used as their original steady-state values, including countries’ output obtained from the Penn World Table (PWT) and net foreign asset positions from the World Bank. The values of the ROW’s variables are the difference between the world aggregate values and those of the countries in the sample.

B.1 Bilateral Trade and Financial Shares

Cross-country trade data are obtained from the Direction of Trade Statistics (DOTS) compiled by the IMF. I use the bilateral import (CIF) data to calculate a country’s spending on goods sourced from other countries. A country’s spending on its own goods is computed as the difference between its gross expenditure and total imports, both available from the World Development Indicators (WDI) compiled by the World Bank.

Financial data are sourced from Factset/Lionshare, a dataset that provides information on institutional investors’ asset holdings. It has comprehensive coverage of institutional holdings across countries. I describe its details in Hu (2022) and its consistency in terms of portfolio composition with macro-level datasets such as IMF’s International Financial Statistics. Factset/Lionshare compiles financial investment by investors’ origin and their investment destination, using which I calculate bilateral portfolio shares.

As the analysis in this paper covers two channels, the sample of countries includes those with trade and finance data both available (see table B.1).

B.2 Productivity

The estimation of productivity consistent with the Eaton and Kortum (2002) model is modified from the approach developed by Levchenko and Zhang (2014), who infer Ricardian productivity from bilateral trade data.

Let any country’s production cost be denoted as

\[ c_{i,t} = (\frac{\mu_i}{\mu_i + u_{i,t}^{1-\mu}})^{\eta} p_{i,t}^{1-\eta}. \]  

(B.1)

\[ \text{23The selection of the sample period is mainly driven by the availability of financial data from Factset.} \]
Table B.1: List of Sample Countries

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It follows from equation 4 that trade shares for any destination country \( j \) should satisfy

\[
\frac{\pi_{ij,t}}{\pi_{jj,t}} = \frac{T_{i,t}}{T_{j,t}} \left( \frac{\tau_{ij,t} c_{i,t}}{c_{j,t}} \right)^{-\theta}. \tag{B.2}
\]

As the left hand side is directly observable from the trade data, we can recover relative productivity \( \frac{T_{i,t}}{T_{j,t}} \) after estimating bilateral trade friction \( \tau_{ij,t} \) and relative input cost \( \frac{c_{i,t}}{c_{j,t}} \).

I follow the trade literature by estimating bilateral trade costs \( \hat{\tau}_{ij,t} \) from a combination of gravity variables including geographic distance divided into intervals,\(^{24}\) dummies for contiguity, common language, common colonizer, common religion, common legal system, and regional trade agreements. The values of these gravity variables are obtained from the CEPII.

I estimate a country’s production cost (denoted as \( \hat{c}_{i,t} \)) based on the information from the PWT. Specifically, I compute a country’s wage \( (w) \) as the ratio of its total labor compensation (output-side GDP \( (rgdpo) \times \) share of labor compensation in GDP \( (labsh) \)) to total labor hours (number of employees \( (emp) \times \) average hours per employee \( (avc) \)). Price of domestic absorption \( (pl_{da}) \) and price of capital services \( (pl_{k}) \) are used as the proxies for the price of intermediate inputs and capital rental fee respectively. Besides, I calibrate the share of intermediate input in production \( \eta = 0.312 \) based on Dekle et al. (2007) and the share of labor input \( 1 - \mu \) as country-specific \( labsh \) from the PWT. The production cost of ROW is calculated as the median cost across countries not included in table B.1.

\(^{24}\)I follow Eaton and Kortum (2002) by setting the distance intervals in miles as [0, 350], [350, 750], [750, 1500], [1500, 3000], [3000, 6000], [6000, \( \infty \)] and code dummies for these intervals as independent variables in the regression analysis.
The full estimating specification for all the country pairs in the sample follows

\[
\ln(\frac{\pi_{ij,t}}{\pi_{jj,t}}) = \ln(T_{i,t} \hat{c}_{i,t}^{-\theta}) - \ln(T_{j,t} \hat{c}_{j,t}^{-\theta}) - \theta \hat{\tau}_{ij,t} + \gamma_{ij,t}, \quad \text{(B.3)}
\]

The first two terms on the right \(\ln(T_{i,t} \hat{c}_{i,t}^{-\theta})\) and \(\ln(T_{j,t} \hat{c}_{j,t}^{-\theta})\) can be captured by the exporter and importer fixed effects respectively when running the estimation. \(\hat{\tau}_{ij,t}\) represents the estimated bilateral trade costs as a linear combination of the gravity variables described above and \(\gamma_{ij,t}\) stands for error terms. Exponentiating the importer fixed effects yields a term that combines country \(j\)'s productivity and cost denoted as

\[
\hat{T}_{c_{j,t}} = T_{j,t} \hat{c}_{j,t}^{-\theta}. \quad \text{(B.4)}
\]

If the US is the benchmark country, and its productivity \((T_{US,t})\) is obtained as the country’s TFP from the PWT \((rtfpma)\). Then other countries’ Ricardian productivity can be calculated as

\[
T_{j,t} = T_{US,t} \frac{\hat{T}_{c_{j,t}}}{\hat{T}_{c_{US,t}}} \left( \frac{\hat{c}_{j,t}}{\hat{c}_{US,t}} \right)^{\theta}, \quad \text{(B.5)}
\]

where trade elasticity is set as \(\theta = 4\) as in Simonovska and Waugh (2014). After calculating countries’ dynamic productivity \(T_{j,t}\), I compute its mean value over time \(\bar{T}\) and the cross-country covariance matrix \(\Sigma_T\). Dynamic productivity with these moments can be used to simulate shocks for intra-regime analysis.