The Pricing of Sovereign Risk Under Costly Information

Grace Weishi Gu\textsuperscript{a}, Zach Stangebye\textsuperscript{b}

\textsuperscript{a} University of California Santa Cruz, grace.gu@ucsc.edu
\textsuperscript{b} University of Notre Dame, zstangeb@nd.edu

January 2017

Abstract

We explore the consequences of costly information acquisition on the pricing of sovereign risk. We consider an environment in which lenders' information acquisition and the sovereign's default decision are jointly endogenous. We find that the model generates state-dependent allocation of investor attention, which has a number of implications: First, it serves as a microfoundation for country-specific time-varying volatility in the spread; second, it implies that model-based estimates of default risk from spread data will be negatively biased during crisis periods; and third, it suggests that some fiscal opacity, as opposed to full transparency, can improve sovereign welfare. We also contribute to the literature by developing an approach to identify the cost of information acquisition for structural model calibrations.

Keywords: costly information, sovereign default, time-varying spread volatility, inference bias, transparency
JEL Codes: F34, D83

About CAFIN

The Center for Analytical Finance (CAFIN) includes a global network of researchers whose aim is to produce cutting edge research with practical applications in the area of finance and financial markets. CAFIN focuses primarily on three critical areas:

- Market Design
- Systemic Risk
- Financial Access

Seed funding for CAFIN has been provided by Dean Sheldon Kamieniecki of the Division of Social Sciences at the University of California, Santa Cruz.
We explore the consequences of costly information acquisition on the pricing of sovereign risk. We consider an environment in which lenders’ information acquisition and the sovereign’s default decision are jointly endogenous. We find that the model generates state-dependent allocation of investor attention, which has a number of implications: First, it serves as a microfoundation for country-specific time-varying volatility in the spread; second, it implies that model-based estimates of default risk from spread data will be negatively biased during crisis periods; and third, it suggests that some fiscal opacity, as opposed to full transparency, can improve sovereign welfare. We also contribute to the literature by developing an approach to identify the cost of information acquisition for structural model calibrations.

Keywords: costly information, sovereign default, time-varying spread volatility, inference bias, transparency.

JEL code: F34 - D83
1. Introduction

Yields in sovereign bond markets in emerging economies largely reflect the risk that a domestic borrower may default on foreign creditors since it does not in principle have any reason to care about the well-being of these creditors. But a sovereign borrower’s lack of welfare concern for its foreign investors is not the only relevant friction that arises from the international nature of these markets: Information frictions also play a large role in cross-border financial transactions: Investors are likely to be less informed about payoff-relevant shocks in other countries (Hatchondo [2004], Van Nieuwerburgh and Veldkamp [2009], or Bacchetta and van Wincoop [2010]). In the information age, information about such shocks can often be acquired, but typically at a cost.

Information frictions are especially significant in the relatively illiquid market for sovereign bonds, in which most transactions occur over-the-counter and knowledge is more disperse.\(^4\) Hence, we seek to understand how this costly information acquisition affects the equilibrium pricing of sovereign default risk. In particular, we construct a model in which the sovereign’s default and borrowing decisions as well as the lenders’ acquisition of payoff-relevant information are jointly endogenous: Lenders can costlessly observe some publicly available states, such as output growth and debt levels, but they cannot directly observe other potentially payoff-relevant shocks when they make investment decisions, such as the severity of a recession implied by a potential future default or populist sentiment in that country. Accurate signals regarding these latter shocks can only be acquired at a cost.

We find that costly information acquisition generates state-dependence in investor attention to publicly observed states. When debt levels are low and output growth is high, investors are aware that there is little to no gain in terms of accurately inferring unobserved shocks since default risk is negligible for most of their realizations. Consequently, they save on information costs and acquire less information. However, for moderately high debt levels and low growth, information is more valuable since the unobserved shocks may substantially affect default risk. Thus, investors are willing to pay more to acquire information about these unobserved shocks.

Intuitively, foreign investors will start poring over more information sources during crises to carefully study the borrower and its default risk, e.g., professional forecasts,

\(^4\)Cole and Kehoe (1998), Sandleris (2008), Catao et al. (2009), and Pouzo and Presno (2015) have all shown that information asymmetries are key to explaining various features of these markets, though none have considered the consequences of allowing information to be gathered at a cost.
IMF staff reports, credit rating agency reports, and public finance records. This has the flavor of practical techniques employed in the financial sector, where fund managers in charge of multiple portfolios pay relatively limited attention to individual country risk unless publicly available indicators in that country trigger some preset alert. At this point, asset managers typically redirect their staff and perhaps take other means to assess more carefully that country’s risk.

Our state-contingent allocation of attention generate four main contributions. First, it serves as a microfoundation for time-varying volatility in the country risk spread, since it generates this feature endogenously while not assuming that any fundamental processes exhibit it. During normal times, investors pay little to no attention to payoff-relevant unobserved shocks. Consequently, they assume that these shocks are at their mean for the purpose of inferring and thus pricing default risk. This implies that bond yields do not respond to realizations of these unobserved shocks during normal times and so spread volatility is lower. During crises, however, investors pay much more attention to these payoff-relevant unobserved shocks. Therefore, they are much better informed about the realization of these shocks that become priced. This implies that bond yields do respond to realizations of unobserved shocks during crisis times, which increases spread volatility. This time-variation in the macroeconomic volatility is a well-documented empirical fact (Justiniano and Primiceri [2008], Bloom [2009], and Fernández-Villaverde et al. [2011]), but little has been done as of yet to understand its causes.\(^5\)

To quantify this channel we develop a model-free metric of time-varying volatility, which we call the Crisis Volatility Ratio (CVR). Comparative statics in the model also suggest that falling information costs should imply a substantial and monotone increase in the CVR. We test this implication in the JP Morgan EMBI database over a period of time in which information costs were arguably falling and find it to be true in nearly every country, with the average percent increase in the CVR being 192.7%.

Second, the state-contingent nature of information acquisition has consequences for econometric inference of default frequencies from spread data. The spread on a short-term sovereign bond can be decomposed into two components: Default risk and a risk premium for that default risk. When information acquisition is costly, the relative contribution of each to the overall spread will change across publicly

\(^5\)Some notable recent exceptions are Seoane (2015) and Johri et al. (2015), but these papers explain time-varying volatility in the country risk spread by assuming exogenous time-varying volatility in fundamentals. ? (??) provides an alternative solution, but it generates time-varying cross-sectional dispersion among firms.
observed states e.g. debt levels and output growth. During non-crisis times, investors do not pay attention because it is costly; the risk premium reflects their ignorance of some potentially payoff-relevant unobserved shocks. When a crisis occurs, however, investors respond to it by mitigating the risk to which they are exposed via greater information acquisition. Thus, while the level of the default risk is greater, investors’ effective risk-aversion is lower.

This implies that during non-crisis times, a relatively larger fraction of the spread is comprised of a risk premium from information uncertainty, and that during crisis times it is mainly comprised of default risk. This has substantial implications for the inference of default risk from spread data, which is a common practice in the literature (Bi and Traum [2012], Bocola [2016], Bocola and Dovis [2016], and Stangebye [2015]). In particular, it implies that a standard sovereign default model will tend to understate default risk during crises by assuming a risk premium that is too large. The risk premium is assumed to be too large because investors have no means by which they can endogenously mitigate their risk exposure through greater information acquisition in response to the crisis.

Third, we explore the trade-offs associated with increasing sovereign transparency by varying information costs. We find a non-monotonicity in sovereign welfare as a function of information costs: The sovereign borrower’s welfare initially increases as information costs fall, but this pattern eventually reverses and it decreases as the costs go to zero. The intuition is simple and operates through debt prices. If there is no transparency i.e. large information costs, then investors demand a greater risk premium during crises, making it more expensive to borrow and service debt precisely when the sovereign is most vulnerable. This risk premium falls as transparency increases, which is consistent with findings in the empirical literature (Kopits and Craig [1998], Poterba and Rueben [1999], Bernoth and Wolff [2008], and Iara and Wolff [2014]). However, under full transparency i.e. free information, a risk-shifting occurs: The sovereign will be fully exposed to the price volatility that results from normally ignored unobserved shocks during normal times. This volatility hurts the risk-averse sovereign. This latter effect has received little to no attention in the literature.

In addition to our results, there is a novelty in our empirical approach: We make use of Google’s Search Volume Intensity (SVI) index to identify the cost of information acquisition, or alternatively the capacity of the channel agents use to process information is novel. The literature has proposed many different measures of infor-
mation acquisition (Barber and Odean [2008], Gervais et al. [2001], and Seasholes and Wu [2007]), but SVI is one of the few direct measures of investor “attention.” Da et al. (2011) demonstrate that it is an effective measure of attention to firm valuation and stocks, but to the authors’ knowledge the index has not yet been used to measure attention to a sovereign nation’s financial position. In fact, most models of ‘rational inattention’ have not used any direct way of inferring this cost (Sims [2003, 2006], Peng and Xiong [2006], and Mackowiak and Wiederholt [2009]). In this paper, we have a strategy to cleanly identify the cost of information: We match the fraction of quarters in which intense attention is paid to the borrower country. If information is infinitely costly, this fraction will be zero; if it is free, this fraction will be one. The attention fraction uniformly increases as information cost decreases; thus, the latter cleanly identifies the former.

Having identified the degree of information costs, we calibrate the model to Ukraine from 2004-2014. First, we find that in the data our spread volatility time-variation measure, CVR, is 3.67, and that the inclusion of costly information acquisition increases the model CVR by 26%, bringing it from 1.68 to 2.12, which is substantially closer to the data. Second, we find the median size of the risk premium discrepancy due to neglecting investors’ time-varying information acquisition behavior to be about 3.4 percentage points, which is about 29% of the inferred default probability. Third, the calibrated level of information costs appear close to the welfare-maximizing level of transparency; a further reduction in information costs would induce so much more price volatility that it would actually hurt the sovereign.

It is important to note that we are restricting attention to information frictions that are country-specific i.e. those that arise from between a single country and its lenders and does not apply globally. This has substantial implications for our results: For instance, one cannot account for all the time-varying volatility by controlling for global metrics such as the CBOE VIX or the P/E ratio, as has been done in the literature (Bocola and Dovis [2016] and Aguiar et al. [2016]); nor can one eliminate the bias in default risk inference using this approach. With regard to the time-varying volatility, our finding corroborates the careful empirical work of Fernández-Villaverde et al. (2011), who find that the bulk of the time-varying interest rate volatility in emerging markets is country-specific rather than global.

Our focus on relations between a single borrower and its lenders over time distinguishes our analysis from the related work of Cole et al. (2016). These authors also explore a model of costly information acquisition in sovereign debt markets, but their
focus is static. They highlight the potential for this channel to cause contagion effects across many countries and generate multiplicity. In contrast, our model highlights the capacity for costly information acquisition to generate time-varying volatility in the country risk spread and act as a potential source of bias in time-series inference.

The remainder of this paper is divided as follows: Section 2 describes the model; Section 3 discusses the data, quantitative implementation of the model, counterfactual analysis, as well as the model’s novel implications; and Section 4 concludes.

2. Model

We consider a small open economy model of endogenous sovereign default in the vein of Eaton and Gersovitz (1981). This is in part for tractability and in part to demonstrate our model’s applicability to the recent, expanding quantitative literature, e.g., Aguiar and Gopinath (2006), Arellano (2008), Hatchondo and Martinez (2009), or Mendoza and Yue (2012). There is a sovereign borrower who issues one-period non-state-contingent debt to a unit mass of foreign lenders. This borrower lacks the ability to commit to repay this debt in subsequent periods and will default if it is optimal to do so ex post.

For clarity, we distinguish a random variable from its realization by having a tilde over the former.

2.1. Sovereign Borrower

2.1.1. Shocks

There are two shocks in this model, both to the sovereign. The first is a growth shock to output that the sovereign receives each period. More specifically, the endowment can be expressed in terms of a sequence of growth rates as follows:

\[ Y_t = Y_0 \times \prod_{s=1}^{t} e^{g_s} \]

where \( Y_0 \) is given. We assume \( g_t \) follows an AR(1) process:

\[ g_t = (1 - \rho) \mu_g + \rho g_{t-1} + \sigma_g \epsilon_t \]

\[ \epsilon_t \]

6 For simplicity of exposition, as the above papers we restrict attention to Markov Perfect Equilibria that can be expressed recursively, though the set of equilibria is potentially much larger (Passadore and Xandri [2015]).
where $\mu_g$ is average growth rate, $\epsilon_t$ is a standard normal, and $\sigma_g$ is the standard deviation of the growth innovation. The endowment and its growth processes are publicly observed.

The second shock in the model is an iid, multiplicative, default cost shock, $\tilde{m}_t$, which applies only in the first period of a default. We assume for simplicity that it is orthogonal to the growth shock, though this assumption is not strictly necessary.\footnote{If $\tilde{m}_t$ shock were correlated with any publicly observed states, then the investors would infer as much information as they could about it from those observed states. After this, they would solve a very similar problem to attain any residual information if it proves valuable to attain. Such a model would behave very similarly to our benchmark, so we do not view this assumption as restrictive.}

This $\tilde{m}_t$ shock is the only shock that is unobserved by foreign investors when they make investment decisions (we will detail the timing below); information regarding it can only be acquired at a cost. It could represent the magnitude of contraction in response to the default, the propensity of other forms of foreign/domestic capital to flow in or out in this event, or the severity of the ensuing international sanctions among other things. We interpret $\tilde{m}_t$ shock as governing the severity of initial output losses at the onset of a default event. In Appendix A, we also show that when $\tilde{m}_t$ shock is log-normally distributed it can alternatively be interpreted as a default preference shock, such as populist sentiment or political uncertainty. In this sense, $\tilde{m}_t$ can stand in for nearly any form of payoff-relevant factors that are not immediately observed by the investors when they make investment decisions.

We collect all publicly observed states, except for the current stock of debt $B_t$, into a vector $s_t$. More specifically, we have $s_t = \{Y_t, g_t\}$, but the intuition behind all of our results goes through with more general publicly known state variables beyond output and growth. As long as there is a distinction between $s_t$ and $m_t$, it does not matter what is actually contained in each set: In any case, investors will condition their propensity to learn about the realizations of $\tilde{m}$ shocks on $s$ and debt levels.

2.1.2. Timing

Timing is important in this model. We assume that sovereign makes default and bond supply decisions at the beginning of period $t$ when the states $s_t = \{Y_t, g_t\}$ realize and are observed. Subsequently, in the middle of period $t$, the $\tilde{m}$ shock that affects $t+1$ period’s payoff is realized, denoted by $m_{t+1}$. The sovereign can observe it perfectly, while the lenders can observe only a noisy signal. After investors make their bids on sovereign bond demand, the sovereign determines a bond price that clears the bond market in period $t$. This timing can be seen in Figure 1.
Notice that we assume that the sovereign cannot change its bond supply when it learns \( m_{t+1} \). This allows us to focus on the role of information acquisition and avoid the complicated and, for our purposes, unnecessary signalling game that would ensue if it could.

### 2.1.3. Bond Supply and Default

As is standard in the literature, we assume a recursive, Markov-Perfect specification with limited commitment on the part of the sovereign. At the beginning of each period, it compares the value of repaying debt, \( V_{R,t} \), with that of default, \( V_{D,t} \), and chooses the option that provides a greater value:

\[
V_t(s_t, B_t, m_t) = \max \{V_{R,t}(s_t, B_t), V_{D,t}(s_t, m_t)\}
\]

Given the timing assumption, we can express the value of repayment at the beginning of period \( t \) as follows:

\[
V_{R,t}(s_t, B_t) = \max_{B_{t+1}} E_{\tilde{m}_{t+1}} \left[ \log \left( C_t(\tilde{m}_{t+1}) \right) + \beta E_{\tilde{s}_{t+1}|s_t} V_{t+1}(\tilde{s}_{t+1}, B_{t+1}, \tilde{m}_{t+1}) \right]
\]

subject to \( C_t(\tilde{m}_{t+1}) = Y_t - B_t + q_t(s_t, B_{t+1}, \tilde{m}_{t+1})B_{t+1} \)

Here the sovereign has time-separable log-preferences over consumption.\(^8\) The determination of the issuance price schedule, \( q_t(s_t, B_{t+1}, \tilde{m}_{t+1}) \), will be discussed in market clearing section below.

\(^8\)Our quantitative results do not rely on the assumption of log-utility. Any concave function will work. The benefit of using log-utility is that the unobserved \( m \) shock can be interpreted either as endowment/supply shocks or as preference/demand shocks.
We assume that when a default happens in period $t$, all debt is wiped out, and in the first period of the default regime the sovereign is subject to the shock $m_t$ realized from period $t-1$ as discussed in the time line. During the entire default regime, the sovereign is excluded from capital markets for a random number of periods and faces persistent output losses. These costs are meant to be interpreted as a tightening of credit conditions, a disruption of trade credit, an imposition of international sanctions, or any other additional cost faced by the sovereign in default. Whereas $m_t$ shock is interpreted as the realized severity to those output losses at the onset of the default event. Under these assumptions, the value of default can be expressed recursively as follows:

$$V_{D,t}(s_t, m_t) = \log(C_t) + \beta E_{\tilde{s}_{t+1}, \tilde{m}_{t+1}|s_t} [\phi V_{t+1}(\tilde{s}_{t+1}, 0, 1) + (1 - \phi)V_{D,t+1}(s_{t+1}, 1)]$$

subject to $C_t = Y_t \times [1 - \psi(g_t)] \times m_t$

where $\psi(g_t)$ is the known persistent cost of a default as a percentage of output. We assume that $\tilde{m}_t$ is log-normally distributed with a mean of 1. Similarly to Arellano (2008) or Chatterjee and Eyigungor (2012), it is allowed to depend on the current growth state. In particular, we assume as those authors do that default is less costly during periods of low growth. $\phi$ is the Poisson rate at which the sovereign regains access to international capital markets.

We define the sovereign's default decision with a binary operator:

$$d_t(m_t, s_t, B_t) = 1\{V_{R,t}(s_t, B_t) < V_{D,t}(s_t, m_t)\}$$

2.2. Foreign Lenders

We assume a unit mass of foreign, risk-averse lenders, similar to Lizarazo (2013) or Aguiar et al. (2016). Lenders arrive in overlapping generations and each lives for two periods. Each lender is endowed with wealth, $w_t$, and solves a portfolio allocation problem, deciding how much to invest in risky sovereign debt and how much to invest in a risk-free asset yielding a return, $r_t$.

When making investment decisions in period $t$, lenders observe $s_t$ and $B_{t+1}$, but not $m_{t+1}$. Nevertheless, lenders do know the marginal distribution of $\tilde{m}_{t+1} \sim N(0, \sigma_{\tilde{m}}^2)$ and receive noisy signals of the shock realization. We denote investor $i$’s signal realization as $x_{i,t+1} = m_{t+1} + \mathcal{E}_{i,t+1}$ where $\mathcal{E}_{i,t+1}$ is the realized noise in the signal and comes from its distribution $\mathcal{E}_{i,t+1} \sim N(0, \sigma_{\mathcal{E}_{t+1}}^2)$ with a chosen $\sigma_{\mathcal{E}_{t+1}}^2$. Lenders can improve the precision of their signals by paying for costly information, which narrows down
Notice that this is isomorphic to choosing $\rho_{mx,t+1} = \text{corr}(m_{t+1}, x_{t+1})$.\footnote{In this model, it is equivalent to solve $\rho_{mx,t+1}$ or $\sigma_{\tilde{E}_{t+1}}^2$. We solve for $\rho_{mx,t+1}$ because it is bounded between 0 and 1.}

We assume that the same cost in acquiring information across investors, thus they all behave the same way, i.e., resorting to the same $\rho_{mx,t+1}$. But they do receive heterogeneous signals and thus they offer different bond demand functions to the sovereign. This is meant to embody the realistic feature that information acquisition is in fact stochastic. Two lenders could exert the same acquisition effort but reach different primary sources that induce each to value the risky debt slightly differently. This dispersion goes to zero as $\rho_{mx,t+1} \to 1$.

For tractability, we separate lenders’ information acquisition problem from their portfolio allocation decisions, calling them Stages I and II respectively. This is similar in spirit to the approach taken by Gabaix (2014) with his two-stage “sparse max” operator. In Appendix B, we discuss the alternative one-stage setup in detail.

In Stage I, all generation-$t$ lenders are ex-ante identical. They can pay to acquire costly information regarding $m_{t+1}$ to reduce their default risk forecast errors. As a result, each of them receives a signal $x_{i,t+1}$ from the distribution of $x_{t+1}$ that whose relevance depends on the chosen $\rho_{mx,t+1}$. We consider $\rho_{mx,t+1}$ as a measure of investor attention. The information acquired is given by a time-invariant function, $I(\rho_{mx,t+1})$, which is increasing in attention $\rho_{mx,t+1}$. In the benchmark, we assume that $I(\cdot)$ is the reduction in entropy in $\tilde{m}_{t+1}$ that comes from knowledge of $x_{t+1}$, but our results do not hinge at all on this functional form.\footnote{This notion of information was developed primarily by Shannon (1958) and applied to economics by Sims (2003, 2006). Here we have $I(\rho_{mx,t+1}) = \frac{1}{2} \log_2 \left( \frac{1}{1 - \rho_{mx,t+1}} \right)$.} Any increasing function would work.

We can formulate lenders’ information acquisition problem as below, given $s_t$ and $B_{t+1}$:

$$
\min_{\rho_{mx,t+1} \in [0,1]} \quad E_{\tilde{E}_{t+1}} E_{\tilde{m}_{t+1}, \tilde{s}_{t+1}} [d_t(\tilde{m}_{t+1}, \tilde{s}_{t+1}, B_{t+1}) - \tilde{d}_{t}]^2 + \kappa I(\rho_{mx,t+1})
$$

subject to

$$
\tilde{d}_t = E_{\tilde{m}_{t+1}, \tilde{s}_{t+1} | s_t} [d_t(\tilde{m}_{t+1}, \tilde{s}_{t+1}, B_{t+1})]
$$

where $d_t(\cdot)$ is the 0/1 binary default identifier.

To see the benefit of information acquisition, notice that the variance of the interior expectation is decreasing in $\rho_{mx,t+1}$. Consequently, the variance in the lenders’ default forecast can be reduced if the lenders are willing to undergo costly information acquisition. When $s_t$ and $B_{t+1}$ indicate greater risk of default, acquisition of
more accurate information will tend to be optimal; when these publicly known states indicate instead that there is little to no default risk, lenders can save on information costs and accept an imprecise or even orthogonal signals ($\rho_{mx,t+1} = 0$).

Once the correlation has been chosen, each lender $i$ in generation-$t$ receives an idiosyncratic signal $x_{i,t+1}$ and then solves the Stage II portfolio allocation problem at period $t$. They consume only in period $t+1$ and exhibit CRRA preferences with a risk preference parameter $\gamma_L$. Therefore, each lender’s bond demand schedule $b_{i,t+1}$ can be determined by solving the following problem for all price level $q \in [0, \frac{1}{1+r}]$:

$$\max_{b_{i,t+1}} E_{\tilde{s}_{t+1},\tilde{m}_{t+1}|s_t,x_{i,t+1}} \left[ \frac{c_{i,t+1}^{1-\gamma_L}}{1-\gamma_L} \right]$$

subject to $c_{i,t+1} = (w_t-b_{i,t+1}q_t)(1+r) + b_{i,t+1}[1-d_t(\tilde{m}_{t+1},\tilde{s}_{t+1},B_{t+1})]$.

Notice each lender’s expectation depends on $\tilde{m}_{t+1}$ shock’s conditional or “updated” distribution that in turn relies on the attention solution from Stage I, $\rho^*_{mx,t+1}$. They are aware of the correlation of their signals with the true $m_{t+1}$, and they use it in an optimal Bayesian forecast. If $\rho^*_{mx,t+1} = 0$, then each lender ignores the signal since she knows it is useless, and the $\tilde{m}_{t+1}$ shock posterior is just its prior. Hence, in this case, each lender demands bond that is flat in $m_{t+1}$ shock realization and her signal and asks for a slightly higher risk premium for the information uncertainty due to the imprecise signal. On the other hand, if $\rho^*_{mx,t+1} > 0$, then each lender’s signal contains some information, and her posterior belief look different from the prior. Hence, each lender’s demand schedule responds to $m_{t+1}$ and her signal, and there is little risk premium that is due to information uncertainty. Therefore, the solution to this problem is a demand schedule $b^*_{i,t+1}(q|s_t,B_{t+1},x_{t+1})$, which is implicitly depends on $\rho^*_{mx,t+1}(s_t,B_{t+1})$. This is at the very heart of the model dynamics, since it implies that (1) investors in our model do respond to the accuracy of the signals and (2) there are some states of the world, i.e. crises, in which signals and, in the aggregate, $m_{t+1}$, are priced, and others in which they are not.

2.3. Market Clearing

After the lenders offer the sovereign their heterogeneous bond demand schedules, the sovereign chooses an issuance price. Notice that upon observing the issuance price, lenders will be able to infer $m_{t+1}$. At this point, however, it is too late for them to make use of this information since all decisions have been taken already.
Naturally, the sovereign chooses the largest price subject to the restriction that it must in fact issue $B_{t+1}$ in the aggregate. We can use this restriction to construct the pricing schedule as follows: For any state $\{m_{t+1}, s_t\}$ and for any issuance $B_{t+1}$, let

$$q_t(s_t, B_{t+1}, m_{t+1}) = \sup \{q \int_{x_{t+1}} b^*_{t,t+1}(q|s_t, B_{t+1}, x_{t+1}) f_{x_{t+1}|m_{t+1}, s_t, B_{t+1}}(x_{t+1}) dx_{t+1} = B_{t+1} \}$$

where $b^*_{t,t+1}$ is the solution from lender $i$'s investment problem in Stage II. Notice that $q_t$ is a function of $\{s_t, B_{t+1}, m_{t+1}\}$ but not $x_{i,t+1}$, because it is based on aggregation of all lenders’ signals. Moreover, $q_t$ is affected by lenders’ attention choice $\rho^*_{m_t}(s_t, B_{t+1})$, since the latter affects not only $b^*_{t,t+1}$ as discussed above but also $f_{x_{t+1}|m_{t+1}}(x_{t+1})$. To see this, since the distribution $f_{x_{t+1}|m_{t+1}}(x_{t+1})$ is log-normal with known mean and variance, when $\rho^*_{m_t}$ is high, the mean depends on the realization of $m_{t+1}$, the market price thus moves with $m_{t+1}$ even though $m_{t+1}$ is unknown to the lenders at the time they offer their bond demand schedules.

2.4. Equilibrium Definition

Having described the model, we can now define our equilibrium:

**Definition 1.** A Markov Perfect Equilibrium is a set of functions,

$\{V_t(s_t, B_t, m_t), V_{R,t}(s_t, B_t), A_t(s_t, B_t), V_{D,t}(s_t, m_t), q(B_{t+1}|s_t, m_{t+1})\}_{t=0}^\infty$ such that

1. $V_{R,t}(s_t, B_t)$ and $V_{D,t}(s_t, m_t)$ solve Recursions 1 and 2 and imply the policy $B_{t+1} = A_t(s_t, B_t)$. Further, $V_t(s_t, B_t, m_t) = \max \{V_{R,t}(s_t, B_t), V_{D,t}(s_t, m_t)\}$.

2. $q(B_{t+1}|s_t, m_{t+1})$ defines aggregate bond demand given by Equation 3 when $d_t(m_t, s_t, B_t) = 1\{V_t(s_t, B_t) < V_{D,t}(s_t, m_t)\}$.

In Appendix C, we demonstrate how this model can be stationarized for solution purposes.

It ought to be noted that as $\sigma_m \to 0$ our model converges to a variant of Aguiar and Gopinath (2006) or Arellano (2008). In this sense ours is a superset of the standard employed in the literature. This limiting result also holds as $\kappa \to 0$ or $\kappa \to \infty$ were we to include the shock $\tilde{m}_{t+1}$ in either of those models.
Given our model setup, in summary, we can endogenously generate acquisition of information that is contingent on observable states. During non-crisis times, information is not particularly valuable to a foreign investor. Thus, investors price sovereign debt assuming unobserved shocks to be at their average. During crisis times, however, information is highly valuable. Consequently, it is both acquired and priced, which increases the volatility of the sovereign spreads. This generates a number of interesting results including time-varying spread volatility, state-contingent risk premia that are relevant for econometric inference, and novel welfare results on transparency for the sovereign borrower.

3. Quantitative Analysis

To determine the impact that costly information acquisition has on the pricing of sovereign risk, we calibrate the model to match a set of empirical moments from Ukraine from 2004-2014. We choose Ukraine since its macroeconomic properties during this time are rather similar to Argentina during the 1990’s, which is the canonical calibration choice for models in this vein (Aguiar and Gopinath [2006] or Arellano [2008]), and thus the model will have little difficulty matching the key moments. In contrast, many Latin American economies during the 2000’s exhibit substantially lower growth and spread volatility than in the decades prior. Further, Ukraine was at the heart of several news cycles over the course of this period, including political upheavals during the Russo-Georgian War in August 2008 and the annexation of Crimea by the Russian Federation in early 2014, which fits especially well with our perceived examples about the unobserved shocks. Last, we choose the period of 2004-2014 because it is the only period for which Google search volume data, which is key in our approach, is available. We solve the model using value function iteration on a discrete grid.

3.1. Data and Calibration

We take data from three primary sources: First is the JP Morgan Emerging Market Bond Index (EMBI) database taken from Datastream; second is the World Bank database; and third is Google Trends’ Search Volume Index (SVI).

3.1.1. Information Cost Identification

First and most importantly, to find a proper cost value per unit information, i.e., $\kappa$, we match the variability of information acquisition in the model and in the
data. Following Da et al. (2011), we measure information acquisition in the data by employing Google search trends for terms for which investors are likely to search. In particular, we consider Google’s *Search Volume Index* (SVI), which is a scalar measure normalized to be always between 0 and 100 for the intensity of Google searches for a given term over a given time period. Zero is for minimum intensity and 100 is for maximum intensity.\(^{11}\)

The process of our information cost identification is as follows. First, we obtain monthly SVI data for 2004-2016 and average them into quarterly data. Then, we transform the raw SVI series into the *Abnormal Search Volume Index* series (ASVI), as suggested by (Da et al. [2011]). The ASVI is meant to capture a notion of paying “extra” attention to a certain event or item in a period \(t\). In any period \(t\), it is computed as follows:

\[
ASVI_t = \log(SVI_t) - \log(\text{Median} \{SVI_{t-9}, SVI_{t-8}, \ldots, SVI_{t-1}\})
\]

In the benchmark, we compute the ASVI for the search term “Ukraine IMF,” since investors are more likely to be interested in relevant IMF staff reports and programs than average search users. We consider other search terms for robustness in the appendix. The full ASVI series can be found in Figure 2. We can see that ASVI lines up well with Ukraine political upheavals. It also slightly leads and shares a positive correlation (43%) with JPMorgan’s EMBI index for Ukraine.

After computing the ASVI, we define an information threshold \(\zeta = 0.5 \times \max \{ASVI_t\}\). Our calibration target then becomes the fraction of quarters when investors acquired information about Ukraine with intensities above this threshold. It captures the frequency of large, positive, and relatively discrete jumps in attention. This is the sort of behavior that our model predicts and so it is our natural target to match.

For our data, the ASVI threshold \(\zeta = 0.55\) and the targeted ratio is 7.1%. It captures the biggest peak in early 2014, which was largely tied to Ukraine’s conflict with Russia and the annexation of the Crimean peninsula.

### 3.1.2. Calibration

To obtain the output process, we estimate via MLE an AR(1) process \((\rho_g = 0.5763, \mu_g = -0.0031, \sigma_g = 0.0256)\) on real hryvnia-valued GDP growth for Ukraine from 2004-2014 at a quarterly frequency.

\(^{11}\)In Appendix D, we show that our metric is highly correlated with another common metric of attention.
We assume that the risk-free rate is fixed at 1% quarterly; that both the sovereign and the lenders exhibit constant relative risk-aversion preferences with CRRA=2, which is standard; and that \( \theta = 0.083 \), which is an estimate used by Mendoza and Yue (2012) for an average duration of 6 years before returning to international bond market.

We choose the form of output costs in autarky similarly to Chatterjee and Eyigungor (2012), with a constant term and a curvature term:

\[
\phi(g) = \phi_0 + \phi_1 \times g
\]

where \( \phi_0 \) and \( \phi_1 \) are both positive. This implies that when growth is negative, the proportional default costs are below \( \phi_0 \) and when growth is positive these costs are above \( \phi_0 \). For simplicity, we also assume that lender wealth is constant over time, i.e., \( w_t = w \).

We calibrate the remaining six parameters, \( \{ \beta, \phi_0, \phi_1, w, \sigma_m, \kappa \} \), using the simulated method of moments (SMM) to target the simulated results from our model at six moments from the corresponding data: Annual default frequency, average debt-service-to-GDP ratio, annual spread volatility, average annual spread, annual spread skewness, and fraction of time in which information acquisition (\( IA \)) is above 50% of
its max value. These parameters are given in Table 1. Our fit is not yet perfect, but it’s very close. Each parameter is mainly identified by its corresponding target moment, though there are cross-partial effects.

Table 1: Calibration by SMM

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target Description</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>$\beta$=0.90</td>
<td>Annual Default Frequency</td>
<td>1.0%</td>
<td>1.1%</td>
</tr>
<tr>
<td>Output cost</td>
<td>$\phi_0$=0.0329</td>
<td>Average Debt-Service-to-GDP Ratio</td>
<td>12.6%</td>
<td>12.0%</td>
</tr>
<tr>
<td>Output cost curvature</td>
<td>$\phi_1$=0.85</td>
<td>Spread Volatility</td>
<td>5.5%</td>
<td>3.9%</td>
</tr>
<tr>
<td>Lender wealth</td>
<td>$w$=1.00</td>
<td>Average Spread</td>
<td>6.4%</td>
<td>5.9%</td>
</tr>
<tr>
<td>Unobs shock std dev</td>
<td>$\sigma_{m}$=0.014</td>
<td>Spread Skewness</td>
<td>2.32</td>
<td>2.42</td>
</tr>
<tr>
<td>Unit info cost</td>
<td>$\kappa$=0.000264</td>
<td>Fraction of Quarters with $IA &gt; \zeta$</td>
<td>7.1%</td>
<td>9.8%</td>
</tr>
</tbody>
</table>

In the following sections, we first examine the properties of the model that are not subject to our specific calibration. We then study the model implications from the quantitative results.

3.2. Model Behavior

Before we go through properties that are unique to this model, we show that along many dimensions it preserves key features of a standard sovereign default model in the literature. For instance, Figure 3 gives the bond demand functions in equilibrium. It exhibits a simple, downward sloping feature that looks remarkably similar to Arellano (2008) despite the added complexity of signal extraction and ex post lender heterogeneity. We can see that better growth shocks lead to higher price schedules, as do worse $m$ shocks to output loss upon default. The magnitude of the former for the same size shock is much larger than the magnitude of the latter. What is interesting about the $m$ shock, though, is that it is state-contingent. In the right panel, for low debt levels, investors do not pay attention to $m$ shocks and thus do not react. Hence, there is no difference across the two price schedules for debt-to-GDP levels lower than about 10% or so. As debt levels rise, though, they begin to pay attention and the schedules begin to diverge.

The equilibrium policy functions are given in Figure 4. We can see that they share the common feature with the quantitative literature that better growth shocks lead to both higher debt levels and faster convergence to those debt levels.

---

12Our model and its results are at quarterly frequency. The model results are adjusted to annual statistics to
Further, Figure 5 provides an event study surrounding a default. Again, the model behaves as a standard model would. We can see that spreads gradually increase prior to a default and exhibit a sharp spike the period before; growth is near or above trend leading up to default, deteriorating gradually at first but then exhibiting a sharp and unexpected drop just prior to the default.

Now, what’s novel in this model is the state-contingent acquisition of information: Investors optimally choose to acquire more information during times of crises i.e. near default. Recall that the ergodic mean of the debt-to-GDP ratio is about 12%. We can see in the policy functions in Figure 6 that for debt levels below this investor attention depends on the underlying growth shock i.e. the observable shock. If growth is high, investors do not pay attention unless debt levels become very large. However, when growth is low, investors pay much attention for levels of debt even well below the ergodic mean. Notice further that for very high debt levels investors cease to pay

match the listed annual targets.
attention as well: This is because default is near certain in these regions, regardless of the realization of the unobserved shocks. Consequently, there is no point paying a cost to learn about those shocks.

**Figure 6: Information Acquisition Policy Function**

In Figure 7, we can see that attention increases during crises, which shows the signal structure leading up to an instance of default. It is clear that signal precision is relatively low 9 quarters prior to the event i.e. during non-crisis times. As publicly known states indicate that default risk is rising, however, investors become more aggressive in their acquisition of information, increasing it from $\rho_{mx} = 0.06$ to $\rho_{mx} =$
0.16 on the cusp of a default event.

**Figure 7:** Information Acquisition Before Default

![Graph showing the state-contingent impact of costly information.](image)

Figure 7 highlights cleanly the state-contingent impact of costly information. It leads us naturally to its implications for the pricing of sovereign risk in the next section.

### 3.3. Novel Implications

The state-contingent acquisition of information has strong effects on some model objects and negligible effects on others. This is shown in Figure 8. We can see that mean debt levels, spreads, and default frequencies are affected only negligibly by the level of information costs. Even though we allow for general equilibrium effects, it seems they are either not present or not strong enough to influence the first moments implied by the model.

But while first moments are relatively unaffected, both higher moments and attention variables are substantially affected by information costs. For instance, Figure 8 (right panel) shows that the fraction of time investors spend paying attention to the sovereign and the spread volatility vary significantly with information costs. The intuition for the former is trivial; the intuition for the latter is that cheaper information means that unobserved $m$ shocks get priced more often instead of being assumed at their average. This mechanically increases spread volatility.
The skewness of the spreads is another moment also effected by our mechanism substantially, as can be seen in Figure 9. The effect is even stronger on this moment than it is on the volatility. This is because exceedingly large spreads are almost always a result of unobserved shocks being both adverse and priced. The more often these shocks are priced, the more often investors are likely to capture instances in which they are adverse.

Given these novel findings from our model, we highlight three results below.
3.3.1. Time-Varying Volatility

The first result is our model endogenizes and amplifies the time-variation in sovereign spread volatility. In the model, spreads exhibit increased volatility during times of crises, since lenders price the unobserved shocks more accurately, rather than assuming them to be at their mean, which they do during non-crisis times to save on information costs. Further, the time-varying volatility is strongly countercyclical and positively correlated with spread levels. This implies that one could interpret our framework as a microfoundation for such models as Melino and Turnbull (1990) or Fernández-Villaverde et al. (2011).

One would be right to question, however, how much of this time-variation in volatility is actually due to the information friction we have. Part of the increase in the volatility prior to a default event comes mechanically from the fact that spreads will be significantly higher prior to a default. To measure our model’s amplification for the time-variation of spread volatility from the information friction (or relative to a standard sovereign default model), we propose an alternative, model-free metric of the time-variation, which we call the Crisis Volatility Ratio or CVR. It is defined as follows: In a series of data, either simulated or empirical, let $\hat{T}$ denote the set of all periods in which the change in the spread from the prior period is above the 95 percentile of its distribution. We call such events “spread crises,” and by construction they are 20× more likely to happen than default events so that we can have enough data points to study. With this notation, we define the CVR as

$$CVR = \frac{1}{|\hat{T}|} \sum_{t \in \hat{T}} \frac{\hat{\sigma}_{t:t+5}}{\hat{\sigma}_{t-6:t-1}}$$

where $\hat{\sigma}_{xy}$ is the sample standard deviation calculated using the periods from $x$ to $y$. This ratio compares the spread volatility for 5 periods immediately prior to a spread crisis and that for 5 periods after, excluding the spread crisis event itself (from $t - 1$ to $t$). If it is larger than one, then crisis periods tend to be more volatile than non-crisis periods.

We compute the CVR for the data and two different model scenarios: Our benchmark model and a model with infinitely costly information. The results are given in Table 2. We can see that time-varying volatility is a strong feature of the data; much stronger than a standard model without information frictions would predict.\(^{13}\) Our

\(^{13}\)The no-information counterfactual here is the CVR produced by the following exercise: Hold fixed the equilibrium
model generates a 29% increase in the CVR, which brings it closer to the data.

Table 2: Crisis Volatility Ratios

<table>
<thead>
<tr>
<th>Data (Ukraine)</th>
<th>Benchmark Model</th>
<th>No-Information Counterfactual</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.67</td>
<td>2.12</td>
<td>1.68</td>
</tr>
</tbody>
</table>

It is worth noting that there is some time-variation in volatility in the model even without information frictions. This is simply because even those publicly known shocks’ fluctuations are not always relevant for default risk and consequently are not always priced. This implies lower price volatility in normal times than crises. This is true in any model of endogenous sovereign default and, to our knowledge, is a previous undocumented finding. Nevertheless, our model’s ability to amplify this underlying the time-variation in spread volatility is substantial and noteworthy.

While our calibrated model puts the degree of amplification at 29%, comparative statics suggest that it could be much larger. Figure 10 gives the percent increase from the same counterfactual for a wide range of different information costs. It makes clear that the degree of volatility time-variation is monotonically decreasing the information costs despite the potential for general equilibrium effects, and that CVR is highly sensitive to information costs. Moving from the benchmark to slightly lower levels of information costs bring the degree of amplification from 29% to near 50% and then to near 100% in the full-information model.

The variation in the CVR across information costs implies an interesting and testable hypothesis. Since the IT revolution, it is relatively undebatable that information costs have been falling over the past 30 years. Our model suggests that this should imply more time-variation in spread volatility from data across the world. We test this prediction in JPMorgan EMBI data and find it to be true. In particular, we compute the percentage increases in the CVRs for every country in the EMBI database between the time periods 1994-2003 and 2004-2014. We find the average of these percentage increases to be 192.7%, which is significant and consistent with our model prediction, which suggests that the CVR could more than double with relatively small changes in information costs. In fact, there are the only two countries (Colombia and Brazil) in the entire dataset of 17 countries to exhibit a falling CVR.

\[\text{policy and default functions i.e. the risk to the lenders, and re-solve the lenders’ problem assuming infinitely costly information. This provides an alternative, non-equilibrium price schedule that describes how a lender with no access to unobserved information would price the default risk. We compute an alternative sequence of spreads in the simulated data with this pricing schedule and use that to compute the counterfactual CVR.}\]
These cross-country CVR results are not driven by the financial crisis that affected most of the global economy in 2008. While it’s true that volatility increased in many developed nations, the country-risk spread in emerging markets actually saw a substantial decrease in spread volatility over these time periods, with the average percentage change being $-14.5\%$. This could be for many different reasons, from successful implementation of reforms to unconventional monetary policy in developed economies, which we do not need to take a stance on why exactly in this paper. What is important, however, is that while those countries’ volatility has fallen over this time period, it has become much, much more time-varying, which suggests strongly that our mechanism is at work consistently with the data in not just Ukraine, but many different emerging/developing countries.

3.3.2. Default-Risk Inference

The second result we highlight is that the risk composition of spreads is not the same during crisis times and non-crisis times. To understand this, first note that we can intuitively break the spread on sovereign debt into four categories: default risks from observed and unobserved shocks, as well as risk premium due to those two types
of shocks.

\[ \text{Spread}_t = \text{Observed Default Risk}_t + \text{Unobserved Default Risk}_t + \]
\[ \text{Observed Risk Premium}_t + \text{Unobserved Risk Premium}_t \]

A standard sovereign default model has only the first two components for its spread, whereas our model has all four. In particular, the fourth component varies across states depending on the lenders’ endogenous information set. When lenders pay the cost to observe normally unobserved states, they learn more about the realization of those shocks. Hence, they can better assess that risk and the unobserved risk premium decreases.\(^\text{14}\)

What does this imply for default risk inference? While default risk is high during a crisis, so too is the lenders’ willingness to learn and contain that risk (i.e., their effective risk-aversion is lower), since they are acquiring more precise signals about unobserved shocks. This implies that the unobserved risk premium comprises a relatively smaller share of the spread during a crisis than during normal times. Consequently, if an econometrician were not to take this into account, instead employing a standard sovereign default model with constant investor attention to infer default risk from spread data, she would underestimate default risk during crises: She would assume the risk premium to be higher than it actually.

Furthermore, our model allows us to quantify the bias if one does not consider the time-varying unobserved risk premium. In particular, we construct an artificial, non-equilibrium no-information price schedule as we did in the previous subsection, and compare its simulated spreads with our benchmark model’s simulated spreads approaching to crisis events. Their differences are captured in Figure 11. It shows a median underestimation of nearly 30% of true default risk during spread crises, equivalent to over 300 basis points. The figure also reveals that there is no bias in normal times, since the bias arises as a result of lenders’ endogenous response to crisis events.

What is also interesting about these consequences of costly information acquisition is that they are country-specific. Thus, they cannot be controlled for using global metrics, such as the CBOE VIX or the P/E ratio, as is often done (Aguiar et al. [2016] or Bocola and Dovis [2016]). Rather, our theory suggests that in order to accurately

\(^\text{14}\)Note that this says nothing about what happens to the level of the spread, which could increase or decrease since the default probability could scale up or down as new information is acquired.
assess default risk, some metric of investor attention, such as SVI or ASVI, must be controlled for.

3.3.3. Transparency

The last result we highlight in this paper has to do with the benefits and costs of transparency in light of our model mechanism. Costly information acquisition can be interpreted in many ways in the context of our model. Up to this point, we have focused on the interpretation that it is difficult for lenders to acquire information about a sovereign. However, the unit information cost $\kappa$ in our framework could also be understood as scaling the level of transparency that a sovereign has about its own domestic affairs and finances. This way, our model can also offer some interesting insights for transparency policies.

Complementary to the common belief among policymakers that improved transparency is full of benefits for a sovereign, our model suggests that transparency is a double-edged sword. When there is zero transparency, i.e., information is infinitely costly, lenders always demand a risk premium for the unobserved shocks, especially during crisis times. This makes it more expensive for the sovereign to borrow and service debt precisely when its marginal utility is the highest. Having more transparency will benefit the sovereign by lowering the unobserved risk premium. However,
when the sovereign is fully transparent, although the unobserved risk premium dis-appears, it is replaced by substantial spread volatility, since now those used-to-be unobserved shocks are constantly priced. This will hurt the risk-averse sovereign as well. Therefore, transparency brings about a risk-shifting: There is the benefit of lower risk premia, but also the cost of higher volatility.

Figure 12: Sovereign Welfare Across Information Costs

To illustrate the above insight, we show the sovereign’s welfare levels along different information costs in Figure 12. We can see that as information cost decreases from the highest end, the sovereign’s welfare first increases; but once the cost lowers to a certain range, its welfare flattens and eventually decreases sharply. This is precisely because of the benefit-cost tradeoff associated with transparency. In fact, the model suggests that the benchmark calibration implies a level of information costs that is close to welfare-maximizing in our simple setup. This result certainly does not imply an abandon of transparency policies nor suggest an optimal policy solution. Our model is kept concise to study the pure mechanism of investors’ state-contingent attention and its implications on sovereign spreads and related. Here, we just emphasize that one of these implications do point out not only benefits but also costs of transparency to emerging/developing countries.
4. Conclusion

In this paper, we explore the consequences of costly information acquisition on the pricing of sovereign risk. We construct a structural model of endogenous default and information acquisition, and calibrate that model to match data targets, in particular, the first three moments of spreads and Google SVI.

We demonstrate that costly information acquisition can generate country-specific time-varying volatility in sovereign bond spread; imply a misspecification bias in estimating default probabilities in standard econometric inference of default risk from yield data; and generate both benefits and costs for emerging/developing countries’ welfare with regard to transparency. We also lay the groundwork for information cost identification strategy through relevant attention metrics.

Possible extensions to our framework could include rollover crises in the vein of Cole and Kehoe (1996), long-maturity debt (Hatchondo and Martinez [2009] or Chatterjee and Eyigungor [2012]), or persistent unobserved shock processes. The intuition of our results would not change with any of these extensions, though the quantitative results may be affected.

This paper also provide a structural foundation for further econometric work in the identification of sovereign risk. In particular, it emphasizes the significance to account for the negative bias that results in default probability estimates during high-risk periods and sovereign debt crises.
References


Appendix A. Proof of Isomorphism Between Preference and Cost Shocks

Notice that under log-preferences, we can express the value of default as

\[ V_{D,t}(s_t, m_t) = \log(Y_t \times [1 - \psi(g_t)]) + \log(m_t) + \beta \mathbb{E}_{\tilde{s}_{t+1}, \tilde{m}_{t+1}|s_t} [\phi V_t(\tilde{s}_{t+1}, 0, 1) + (1 - \phi) V_{D,t}(\tilde{s}_{t+1}, 1)] \]

which implies that we can define \( \hat{V}_{D,t}(s_t) = V_{D,t}(s_t, m_t) - \log(m_t) \), where \( \hat{V}_{D,t}(s_t) \) is the value of default when the only cost of defaulting is \( \phi(g_t) \).

With this notation, the default decision can be written as

\[ d_t(m_t, s_t, B_t) = 1\{V_{R,t}(s_t, B_t) < \hat{V}_{D,t}(s_t) + \log(m_t)\} \]

In this isomorphic environment, \( m_t \) becomes a preference shock to the benefit of defaulting.

Appendix B. Discussion of Lender Decision Stage

Our two-stage approach to the lenders’ problem is novel, tractable, and, as it turns out, quite necessary. To see why, consider the alternative approach, in which lenders choose their attention \( \rho_{mx,t+1} \) to maximize their expected utility of consumption rather than to mitigate default risk forecast errors. In this environment, strategic complementarities emerge that not only render the model intractable but also are tangential to its goal to study the impact of collective investor attention on sovereign spread dynamics.

The complementarities arise in this alternative environment from two sources: First, lenders must forecast the equilibrium price; and second, lenders seek to minimize risk. Consider the incentives of an individual lender when all other lenders are choosing not to investigate and there is little immediate observed default risk. In this case, the market price will not reflect fluctuations in \( m_{t+1} \) and thus there is no need to pay attention. Since it is costly to pay attention, an individual lender would choose not to do so.

Now consider the same observed default risk but suppose that all other lenders are paying attention. In this case, the price will no longer be flat in \( m_{t+1} \). An individual investor now will concern not about unobserved default risk directly, but rather price
risk. She will pay attention to mitigate this latter risk. Thus, in some states of the world, it could well be the case that if all other lenders are paying attention, then an individual lender is induced to pay attention too; but if all other lenders are not paying attention, then neither will the individual lender.

This form of multiplicity is slightly different than that found in Cole et al. (2016), who show that information acquisition in a simple, finite-horizon model with multiple countries can induce multiple equilibria. In their model, investors pay attention when other investors pay attention to prevent overpaying for debt in bad states of the world and to gain the capacity to offer a menu of quantities. But here, individual investors pay attention to mitigate the price risk that arises from the information acquisition choice.

The sorts of “news panics” that arise here can yield convergence problems in the numerical solution of the model, since some states of the world could hop between various forms of these static equilibria. Even if we could find a solution, however, it’s not clear that comparative statics would be meaningful, since a slight change in parameters could also result in a substantially different equilibrium. Moreover, these sorts of panics are what this paper intends to focus on and distract us from the interesting and novel results we derive in the benchmark model. Hence, we employ our two-stage approach.

Appendix C. Solving the Model

We consider the more general case of CRRA preferences for simplicity of exposition. We stationarize this model by dividing the sovereign resource constraint in every period by $Y_t$, and the value functions by $Y_t^{1-\gamma}$. This delivers a convenient recursive structure which is independent of both time and the level of output. We denote $b' = B_{t+1}/Y_t$ and $c = C_t/Y_t.$

$$v_R(g, b) = \max_{b' \geq 0} E_{\tilde{m}} \left[ \frac{c(\tilde{m})^{1-\gamma}}{1-\gamma} + \beta E_{g | g} E_{\tilde{g}}^{1-\gamma} v(\tilde{g}, b', \tilde{m}') \right]$$

s.t.  $c(\tilde{m}) = 1 - be^{-g} + q(g, b', \tilde{m}')b'$

\footnote{It’s not even clear that we could use a sunspot to coordinate these, as is done in Cole and Kehoe (1996), since we do not know how many such static equilibria may arise and in which states they would arise.}
The value of default is scaled similarly, yielding
\[ v_D(g, m) = \frac{[1 - \psi(g)] \times m}{}^{1-\gamma} + \beta E_{g, \tilde{\gamma}, \tilde{m}} \left[ \phi e^{\tilde{\gamma}(1-\gamma)} v(\tilde{g}', 0, 1) + (1 - \phi) e^{\tilde{\gamma}(1-\gamma)} v_D(\tilde{g}', 1) \right] \]

This stationarization implies that we can express the default policy function using only stationarized model objects, since \( Y_t \) does not influence the default decision once \( g_t \) is known.

\[ d(m, g, b) = 1 \{ e^{-g(1-\gamma)} v_R(g, b) < e^{-g(1-\gamma)} v_D(g, m) \} \]

The benchmark model with sovereign’s log-preferences will simply be the limiting case as \( \gamma \to 1 \). This will imply that the stationarized model will feature log flow utility and that the impact of \( \tilde{g} \) on the effective discount factor vanishes.

With this simplification, we can stationarize the lenders’ problem as well:

\[
\min_{\rho, mx \in [0, 1]} E_{\tilde{\gamma}, \tilde{m}', \tilde{g}'|x', g} \left[ d(\tilde{m}, \tilde{g}, \tilde{g}') - d \right]^2 + \kappa I(\rho_{mx})
\]

The second stage of the lenders’ problem can also be stationarized as well under the assumption that \( w_t = wY_t \).

\[
\max_{\rho'} E_{\tilde{\gamma}', \tilde{g}'|x', g} \left[ \frac{\rho'^{1-\gamma_L}}{1-\gamma_L} \right]
\]

s.t. \( \rho' = (w - b'_t q)(1 + r) + b'_t [1 - d(\tilde{m}', \tilde{g}', b')] \)

**Appendix D. Robustness: Alternative Measure of Attention**

In this section we explore the correlation of our chosen attention metric, SVI, with another common metric of investor attention in the literature, extreme daily returns (Barber and Odean [2008]). In particular, we use daily return data on the stripped sovereign spread from JP Morgan’s EMBI database and compute the largest monthly return in absolute value. We then compare this series to the monthly SVI series in Figure D.1. While they do not line up perfectly, there is much co-movement (\( \rho = 0.3541 \)) and thus our metric lines up reasonably well with this other proxy.

Most of this co-movement is driven by extreme negative returns, which is consistent with our theory since investors pay attention more during crises. The correlation with
SVI and the minimum daily return in a month is $-0.5208$, which provides even further evidence of the relevance of our metric.

Another alternative considered in the literature are daily trade volumes. However, such data is difficult to acquire and may not exist for sovereign bonds since, for Ukraine at least, almost all trades are executed over-the-counter instead of in an exchange.