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A Dynamic Model of Firm Valuation

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We propose a dynamic version of the dividend discount model, solve it in closed-form, and assess its empirical validity. The valuation method is tractable and can be easily implemented. We find that our model produces equity value forecasts that are, on average, very close to market prices, and explains a large proportion (around 83%) of the observed variation in share prices. Moreover, we find that a simple portfolio strategy based on the difference between market and estimated values earns considerably positive returns, on average. These returns are uncorrelated with the three risk factors in Fama and French (1993).

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1 Introduction

We derive a dynamic model of the firm in closed-form and show that it can be used for actual firm valuation. To test its empirical validity, we price firms included in the S&P 100 Index in the period 1990-2015 and evaluate the results from different perspectives. First, we find that the model produces consistent forecasts of stock prices in the sense that model predicted values are very close to the actual market values, on average. In addition, the model explains a large fraction (around 83%) of the variation in current market prices. Second, we also find that the temporary or short-run deviations between market prices and model estimates can be economically exploited. Overall, we believe these results suggest our model is a promising pricing tool that may enhance current approaches to firm valuation.

We use dynamic programming to develop a model of the firm in which the former chooses how much to invest, labor, and how to finance its assets in every period. While this type of models have been used extensively in corporate finance to explain firm behavior, we introduce three fundamental features that make our model particularly useful for asset pricing purposes. First, we do not assume agents are risk-neutral. Instead, we invoke the two-fund separation principle, which shows that, as long as we discount future cash flows with an appropriately risk-adjusted discount rate, we do not need any assumption about shareholders' utility functions.[1] Second, we allow the firm to grow in the long-run, which could be interpreted as the firm having the possibility to take advantage of new, profitable investments in the future. In our model, the firm grows at the constant rate of the corresponding industry in every period, making the former better suited for valuing large and mature firms. Third, we introduce risky debt to our model and find an analytic solution, which, to the best of our knowledge, is novel among existing dynamic programming models of the firm. In particular, debt in our model is protected by a positive net-worth covenant and, in the event of bankruptcy, the firm pays the bankruptcy costs, is reorganized under Chapter 11 of the U.S. Bankruptcy Code, and continues its operations. Under these assumptions, our model generates a debt behavior that is in line with the empirical

[1] See, for example, Copeland, Weston, and Shastri (2005) for a more complete discussion of the separation principle. A critical assumption of this theorem is the possibility of shareholders to access well-developed capital markets.

evidence. For instance, survey results from Graham and Harvey (2001) suggest that most firms have a target leverage. Consistently, the firm in our model chooses debt in every period following a target leverage that depends on its own characteristics.

As mentioned above, an important advantage of our model regarding valuation is that we solve it analytically. Closed-form equations are strongly preferred to numerical approximations because the former yield extremely accurate values at very low computing time. Indeed, a usual problem with the numerical solution of dynamic programming models is the so-called Bellman's curse of dimensionality. This problem arises from the discretization of continuous state and decision variables, since the computer time and space needed increases exponentially with the number of points in the discretization (Rust, 1997, 2008). Thus, more accurate firm valuations imply necessarily exponentially longer periods of computing time. In addition, explicit solutions allow the user to estimate model parameters with ease.

We start analyzing the performance of our model by doing a comparative statics analysis of the stock price with respect to all model parameters. We find that share price is more sensitive with respect to the operating aspects of the firm (e.g., the curvature of the production function and the persistence of profit shocks). This information helps the user to, for instance, ascertain which parameters require greater attention in the estimation step. We then proceed to study the actual pricing performance of the model with firms that were included in the S&P 100 Index in the period 1990-2015. We first compute the ratio of the actual market prices to the values predicted by our model and its mean value turns out to be around 1. This result suggests that our model yields equity value estimates that are, on average, very close to market values. We then regress the market value of equity on the value estimated by our model and find that the latter can explain a large fraction of the observed variability of the former (around 83%). This outcome turns out to be better than the results reported by related papers (described below), and implies a strong linkage between model implied and market values over time.[2]

While we show that our estimates are, on average, very close to market values, we also

[2] Complementing the results in this article, Lazzati and Menichini (2015b) show that a simpler version of this model also explains numerous important regularities documented by the empirical literature in corporate finance. For instance, it rationalizes the negative association between profitability and leverage, the existence and characteristics of all-equity firms, and the inverse relation between dividends and investment-cash flow sensitivities.

find temporary deviations between stock prices and model estimates. We then implement simple portfolio strategies to test whether we can take advantage of those deviations. The former consist in ranking firms based on their ratios of market prices to model estimates, then forming quintile portfolios based on those ratios, and finally buying the firms in the lowest quintile portfolio and selling the firms in the highest quintile portfolio. Our results show that those strategies earn, on average, around 21%, 36%, and 52% returns after one, two, and three years of portfolio formation, respectively. We also study whether those returns can be explained by the three risk factors described by Fama and French (1993), but find that they are uncorrelated with the latter. As benchmark, we calculate the returns of portfolios constructed according to two well-known ratios: market-to-book and price-earnings. We find that the former yields around 13%, 19%, and 30% returns after one, two, and three years of portfolio formation, while the latter yields 8%, 14%, and 20% returns in the same periods. That is, our portfolio strategy consistently outperforms those based on the market-to-book and price-earnings ratios.

To do the previous analyses, we use the simulated method of moments to estimate the structural parameters for firms included in the S&P 100 Index during the period 1990-2015. This procedure estimates parameters by minimizing the distance between certain moments computed from the data and the same moments simulated with the model.[3] In all our estimations, we perform a forward-looking exercise in the sense that we use data available prior to the valuation period to make out-of-sample predictions. Doing so is important because this procedure replicates the situation a user would face when performing actual valuation.

Finally, we should also mention that we valuated firms included in the S&P 100 Index in order to assess the empirical performance of our model. However, our valuation model can be implemented with firms for which market prices do not exist, as long as financial statements are available for parameter estimation. These cases include, among others, private companies such as Koch Industries and Cargill, IPOs such as Facebook in 2012 and Alibaba Group Holding in 2014, and firms' new investment projects.

[3]The simulated method of moments is analyzed theoretically in Gourieroux, Monfort, and Renault (1993) and Gourieroux and Monfort (1996). Strebulaev and Whited (2012) contains a detailed description of its implementation.

Literature Review

Our paper contributes to two different strands of literature in finance, namely, dynamic programming models of the firm and firm valuation models.

Several papers in corporate finance use different dynamic programming models of the firm to explain firm choices. For instance, Moyen (2004, 2007), Hennessy and Whited (2005, 2007), Hennessy, Levy, and Whited (2007), Tserlukevich (2008), Riddick and Whited (2009), and Hennessy, Livdan, and Miranda (2010), among others, use those models to rationalize a large number of stylized facts about firm behavior.[4] We show that —after introducing some new features— this type of models can also be used successfully for firm valuation. As we mentioned before, we get rid of any specification of shareholders' utility function by assuming the two-fund separation principle. In the context of the latter, as long as we discount future cash flows with market discount rates, we can disregard the risk-neutrality assumption. This possibility was suggested by Dixit and Pindyck (1994) and we believe our paper is one of the first attempts in this direction. Another important feature regarding valuation is the possibility of the firm to grow in the long-run. As documented by Lazzati and Menichini (2015a), secular growth can account for more than 30% of the value of the firm, and it is of particular importance for certain industries, such as manufacturers of chemical products and industrial machinery, and providers of communication services (Jorgenson and Stiroh, 2000). In addition, as we explained before, the fact that we obtain closed-form solutions is very important for the accuracy of the model predictions.

While we show in this paper that our model produces successful valuation results, we also find that it performs similarly in some regards and better in some others when compared to other valuation models. We obtain similar results to those of Kaplan and Ruback (1995) and Copeland, Weston, and Shastri (2005), who implement the discounted cash flow model (DCFM) and show that it produces value forecasts that are, as in our case, roughly equal to market prices. With respect to the explanation of the variation in current market prices, our model seems to outperform the results in some related studies: Bernard (1995) compares the ability of the dividend discount model (DDM) and the residual income model (RIM) to explain the observed variation in stock prices. He finds that the RIM explains 68% of the variability in

[4] See Strebulaev and Whited (2012) for a comprehensive review of this literature.

market values and outperforms the DDM, which can only explain 29% of such variation.[5] In a similar study, Frankel and Lee (1998) test the RIM empirically and find that the model estimates explain around 67% of the variability in current stock prices. More recently, Spiegel and Tookes (2013) use a dynamic model of oligopolistic competition to perform cross-sectional valuation and find that their model explains around 43% of the variation in market values. Compared to these papers, we find that our model can explain a higher fraction of the variability of the stock prices (around 83%).[6]

The paper is organized as follows. In Section 2, we derive a dynamic version of the DDM in closed-form and explain its main parts. Section 3 contains the sensitivity analysis of the stock price with respect to the different firm characteristics. The empirical evaluation of the performance of our model is in Section 4. Section 5 concludes. Appendix 1 contains the proofs.

2 A Dynamic Dividend Discount Model

In this section, we derive a dynamic version of the standard DDM in closed-form. We solve the problem of the firm (i.e., share price maximization) using discrete-time, infinite-horizon, stochastic dynamic programming. The solution is obtained within the context of the Adjusted Present Value (APV) method introduced by Myers (1974), which has been used extensively with dynamic models of the firm (e.g., Leland, 1994; Goldstein, Ju, and Leland, 2001; and Strebulaev, 2007).[7]

2.1 The Problem of the Firm

The life horizon of the firm is infinite, which implies that shareholders believe it will run forever. The CEO makes investment, labor, and financing decisions at the end of every time

[5] The DDM, the DCFM, and the RIM are theoretically equivalent, but they differ with respect to the information used in their practical implementation. The DDM uses the future stream of expected dividend payments to shareholders. The DCFM is based on some measure of future cash flows, such as free cash flows. Finally, the RIM uses accounting data (e.g., current and future book value of equity and earnings).

[6] In all those papers, the samples differ in terms of firm composition and time periods.

[7] As it is common with other valuation models (e.g., the Black-Scholes formula), we do not introduce transaction or adjustment costs to our model.

period (e.g., month, quarter, or year) such that the market value of equity is maximized. (In this paper, we write a tilde on X (i.e., $\widetilde{X}$) to indicate that the variable is growing over time.) Variable $\widetilde{K}_t$ represents the book value of assets while variable $\widetilde{L}_t$ indicates the amount of labor used by the firm in period t. In each period, installed capital depreciates at constant rate $\delta > 0$.

The debt of the firm in period t, $\widetilde{D}_t$, matures in one period and is rolled over at the end of every period. We assume debt is issued at par by letting the coupon rate c_B equal the market cost of debt r_B. In turn, this feature implies that book value of debt $\widetilde{D}_t$ equals the market value of debt $\widetilde{B}_t$. The amount of outstanding debt $\widetilde{B}_t$ will increase or decrease over time according to financing decisions. We let debt be risky, which implies that the firm goes into bankruptcy when profits are sufficiently low (e.g., negative). Following Brennan and Schwartz (1984), we assume the debt contract includes a protective covenant consisting of a weakly positive net-worth restriction. According to that covenant, in order to avoid bankruptcy, the book value of equity must be weakly positive. In the event of bankruptcy, the firm has to pay bankruptcy costs $\xi\widetilde{K}_t$ (with $\xi > 0$), such as lawyer fees and other costs of the bankruptcy proceedings. Furthermore, we assume the bankrupt firm is reorganized and continues its operations after filing for protection under Chapter 11 of the U.S. Bankruptcy Code. This assumption is consistent with the empirical evidence showing that the majority of firms emerge from Chapter 11 and only few are actually liquidated under Chapter 7 (see, e.g., Morse and Shaw, 1988; Weiss, 1990; and Gilson, John and Lang, 1990). Finally, as in Hennessy and Whited (2007), we let those costs be proportional to the level of assets.

We introduce randomness into the model through the profit shock z_t. Following Fama and French (2000), we let profits be mean-reverting by assuming that random shocks follow an AR(1) process in logs

$$\ln(z_t) = \ln(c) + \rho \ln(z_{t-1}) + \sigma x_t \tag{1}$$

where $\rho \in (0, 1)$ is the autoregressive parameter that defines the persistence of profit shocks. In other words, a high ρ makes periods of high profit innovations (e.g., economic expansions) and low profit shocks (e.g., recessions) last more on average, and vice versa. The innovation term x_t is assumed to be an *iid* standard normal random variable, scaled by constant $\sigma > 0$. The latter defines the volatility of profits over time. Finally, constant $c > 0$ defines the mean value toward which random shocks revert (i.e., $E[\ln(z_t)] = \ln(c)/(1-\rho)$). Thus, this parameter has a direct impact on expected profits and an important influence on the current size of the firm. This parameter is important because it helps to capture unobserved firm-specific features affecting firm profitability and are not explained by either ρ or σ. It is common in the corporate finance literature to normalize c to 1, which allows for the study of representative firms. However, that assumption undermines the valuation attempt as the estimated prices may result strongly biased. Given that our objective is to evaluate the pricing performance of our model with actual data, we do not do such normalization and use our modeling restrictions to recover the value of c for each firm.

Gross profits in period t are defined by the following function

$$\widetilde{Q}_t = (1+g)^{t[1-(\alpha_K+\alpha_L)]} z_t \widetilde{K}_t^{\alpha_K} \widetilde{L}_t^{\alpha_L} \tag{2}$$

where z_t is the profit shock in period t, $\alpha_K \in (0,1)$ represents the elasticity of capital, and $\alpha_L \in (0,1)$ indicates the elasticity of labor (we further assume $\alpha_K + \alpha_L < 1$). Constant g represents the growth rate of the industry to which the firm belongs and, with this factor, profits, costs and firm size grow proportionately over time.[8] Caves (1998) finds that firm growth becomes less volatile over time as the firm becomes large. Therefore, our restriction makes the model better suited for pricing lower growth firms, such as the large and mature corporations we value in this article (i.e., firms in the S&P 100 Index). Equation (2) says that gross profits also depend on a Cobb-Douglas production function with decreasing returns to scale in capital and labor inputs.[9]

Every period, the firm pays operating costs $f\widetilde{K}_t$ (with $f > 0$) and labor wages $\omega\widetilde{L}_t$ (with $\omega > 0$), while corporate earnings are taxed at rate $\tau \in (0,1)$. Therefore, the firm's net profits in period t are

$$\widetilde{N}_t = \left(\widetilde{Q}_t - f\widetilde{K}_t - \delta\widetilde{K}_t - \omega\widetilde{L}_t - r_B\widetilde{B}_t\right)(1-\tau). \tag{3}$$

With all the previous information, we can state the cash flow that the firm pays to equity-holders

[8] Factor $(1+g)^{t[1-(\alpha_K+\alpha_L)]}$ allows us to use a standard normalization of growing variables that is required to solve the problem of the firm (see, e.g., Manzano, Perez, and Ruiz, 2005).

[9] Equation (2) can take on only (weakly) positive values. However, the model can be easily extended to allow for negative values of gross profits by subtracting a positive constant as a proportion of assets (e.g., $a\widetilde{K}_t$) in equation (2).

in period t as

$$\widetilde{Y}_t = \widetilde{N}_t - \left[\left(\widetilde{K}_{t+1} - \widetilde{K}_t\right) - \left(\widetilde{B}_{t+1} - \widetilde{B}_t\right)\right] - \Theta\xi\widetilde{K}_t. \tag{4}$$

According to equation (4), the dividend paid to shareholders in period t equals net profits minus the change in equity and, in the event of bankruptcy (i.e., $\Theta = 1$), minus the bankruptcy costs. We let rate r_S represent the market cost of equity and rate r_A denote the market cost of capital. We also assume the secular growth rate is lower than the market cost of capital (i.e., $g < r_A$). Finally, given the current state at $t = 0$, (K_0, L_0, B_0, z_0), the problem of the firm is to make the optimal capital, labor, and debt decisions, such that the market value of equity is maximized.[10]

Before we solve the firm problem, we need to convert it into stationary, for which we normalize the growing variables by the gross growth rate: $X_t = \widetilde{X}_t/(1+g)^t$, with $X_t = \{K_t, L_t, B_t, Q_t, N_t, Y_t\}$. We let E_0 indicate the expectation operator given information at $t = 0$. Using the normalized variables and modifying the payoff function accordingly, the market value of equity can be expressed as

$$S\left(K_0, L_0, B_0, z_0\right) = \max_{\{K_{t+1}, L_{t+1}, B_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \frac{(1+g)^t}{\prod_{j=0}^{t}\left(1 + r_{S_j}\right)} Y_t. \tag{5}$$

We solve that problem by separating investment (and labor) from financing decisions, as shown by Modigliani and Miller (1958). This separation is possible in our dynamic model because debt is a static decision (i.e., it can be fully adjusted in each period of time).

2.2 Model Solution

To help streamline the exposition, we solve the problem of the firm —equation (5)— in Appendix 1. We find the following closed-forms for the stock price when the firm does not go into bankruptcy. As a by-product, we also characterize the probability of bankruptcy and the optimal leverage ratio analytically.

Proposition 1 *The market value of equity is*

$$\begin{aligned} S\left(K_t, L_t, B_t, z_t\right) = {} & \left[z_t K_t^{\alpha_K} L_t^{\alpha_L} - fK_t - \delta K_t - \omega L_t - r_B B_t\right](1-\tau) + \\ & K_t - B_t + G\left(z_t\right) \end{aligned} \tag{6}$$

[10] The restriction $\left(f + \delta + \omega\frac{\Phi_2^*}{\Phi_1^*}\right)(1-\tau) + \xi \leq 1$ guarantees that both debt and firm value are weakly positive.

where the going concern value is $G_t(z_t) = M(z_t)\, P^*$. *Variable* $M(z_t)$ *is given by*

$$M(z_t) = e^{-\frac{1}{2}\sigma^2 \frac{(\alpha_K+\alpha_L)}{[1-(\alpha_K+\alpha_L)]^2}} \left\{ \left(\tfrac{1+g}{1+r_A}\right) E\left[z_{t+1}^{1/[1-(\alpha_K+\alpha_L)]} | z_t\right] + \left(\tfrac{1+g}{1+r_A}\right)^2 E\left[z_{t+2}^{1/[1-(\alpha_K+\alpha_L)]} | z_t\right] + \ldots \right\} \tag{7}$$

with the general term

$$E\left[z_{t+n}^{1/[1-(\alpha_K+\alpha_L)]} | z_t\right] = \left(c^{\frac{1-\rho^n}{1-\rho}} z_t^{\rho^n} e^{\frac{1}{2}\sigma^2 \frac{\left(1-\rho^{2n}\right)}{(1-\rho^2)} \frac{1}{[1-(\alpha_K+\alpha_L)]}} \right)^{\frac{1}{1-(\alpha_K+\alpha_L)}}, \quad n = 1, 2, \ldots \tag{8}$$

and variable P^* *takes the form*

$$P^* = \left(\Phi_1^{*\alpha_K} \Phi_2^{*\alpha_L} - f\Phi_1^* - \delta\Phi_1^* - \omega\Phi_2^*\right)(1-\tau) - r_A\Phi_1^* + \left(\frac{1+r_A}{1+r_B}\right)(r_B\tau\ell^* - \lambda^*\xi)\,\Phi_1^* \tag{9}$$

with

$$\Phi_1^* = \left[\left(\frac{\alpha_K}{\frac{r_A}{1-\tau} + f + \delta} \right)^{1-\alpha_L} \left(\frac{\alpha_L}{\omega}\right)^{\alpha_L} \right]^{\frac{1}{1-(\alpha_K+\alpha_L)}} \tag{10}$$

and

$$\Phi_2^* = \left[\left(\frac{\alpha_K}{\frac{r_A}{1-\tau} + f + \delta} \right)^{\alpha_K} \left(\frac{\alpha_L}{\omega}\right)^{1-\alpha_K} \right]^{\frac{1}{1-(\alpha_K+\alpha_L)}}. \tag{11}$$

The probability of bankruptcy is

$$\lambda^* = \int_{-\infty}^{x_c^*} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \tag{12}$$

where

$$x_c^* = -\sigma - \sqrt{2\left\{\sigma^2 + \ln\left\{ \frac{\left[1 + \frac{1}{r_B(1-\tau)}\right]\xi}{\sqrt{2\pi}\tau\sigma\Phi_1^{*\alpha_K-1}\Phi_2^{*\alpha_L}} \right\}\right\}}. \tag{13}$$

The optimal book leverage ratio is given by

$$\ell^* = \frac{1 + \left[e^{\sigma\left(x_c^* - \frac{1}{2}\sigma\right)}\Phi_1^{*\alpha_K-1}\Phi_2^{*\alpha_L} - f - \delta - \omega\frac{\Phi_2^*}{\Phi_1^*}\right](1-\tau) - \xi}{1 + r_B(1-\tau)} \tag{14}$$

The market value of equity is shown in equation (6) and represents an analytic solution of the Gordon Growth Model (Gordon, 1962) in the dynamic and stochastic setting. The first three terms in equation (6) represent the after-shock book value of equity, while the last term, $G(z_t)$, is the going-concern value. The latter depends on variable $M(z_t)$, which captures the effect of the infinite sequence of expected profit shocks, and on variable P^*, which denotes the dollar return on capital minus the dollar cost of capital at the optimum (including the interest tax shields and bankruptcy costs as financing side effects).[11] The going-concern value shows that, using only information about the current state (i.e., assets, labor, debt, and gross profits), our model solves systematically for the full sequence of expected future dividends. Our model also projects automatically the value of the real options (such as the option to expand the business, extend the life of current projects, shrink the firm, or even postpone investments) by allowing the firm to optimize decisions over time.[12]

Optimal capital turns out to be $K^*(z_t) = (1+g)\, E\left[z_{t+1}|z_t\right]^{\frac{1}{1-(\alpha_K+\alpha_L)}} \Phi_1^*$, which shows how the former depends on the primitive firm characteristics. As expected, it decreases with the market cost of capital r_A, operating costs f, depreciation δ, and labor costs ω. On the contrary, optimal assets increase with the growth rate g, constant c, and the volatility of innovations σ because they increment the expected profitability of capital via equation (3). The effects of α_K, α_L, and ρ depend on current profit shock z_t, but they are generally positive for standard values of the parameters. Optimal labor has the form $L^*(z_t) = (1+g)\, E\left[z_{t+1}|z_t\right]^{\frac{1}{1-(\alpha_K+\alpha_L)}} \Phi_2^*$, and its sensitivity with respect to the fundamental characteristics of the firm is analogous too that of optimal assets. Optimal debt is $B^*(z_t) = \ell^* K^*(z_t)$ and all the previous characteristics have the same directional effects on this decision. Finally, the income tax rate τ has a negative effect on optimal assets and labor because the latter become less profitable as the former is higher. It also has a negative effect on optimal debt for the great majority of parameter values, including those used in this paper.

In a survey, Graham and Harvey (2001) provide empirical evidence suggesting that most firms actually follow some form of target leverage. Consistently, our model produces an optimal debt that is a constant proportion of optimal assets, with the factor of proportionality given by ℓ^* in equation (14). This optimal ratio can be interpreted as the target leverage of the firm. It is

[11] Function $M(z_t)$ suggests that our model can become an n-stage dynamic DDM if we substitute the growth rate g on the numerator of the discount factor appropriately.

[12] In a simpler version of this model, Lazzati and Menichini (2015a) show that the value of the real options can easily represent more than 8% of the stock price and is particularly important for certain industries, such as in Oil and Gas Extraction. This is a further advantage of our model over static ones.

readily verified that ℓ^* is strictly less than 1 and bounded below by zero, decreases in non-debt tax deductions (e.g., operating costs f and depreciation δ) and the market cost of debt r_B, and is an increasing function of the income tax rate τ. These comparative statics predictions conform with the testable implications of the model.

Finally, our model also yields a constant probability of bankruptcy, which is in line with the findings of Kisgen (2006, 2009), who suggest that firms aim to maintain their debt within certain credit ratings.[13] As expected, this probability increases with the operating costs f, depreciation δ, the market cost of debt r_B, and the income tax rate τ.

3 Sensitivity Analysis of the Stock Price

We now investigate how sensitive the stock price is with respect to the different features of the firm model (e.g., the curvature of the production function, the operating costs, the volatility of profits, etc.). This analysis could guide practitioners in identifying the firm parameters that need to be more carefully calibrated/estimated, as small deviations could have large impacts on the stock price. We do this analysis for representative firms in different SIC industries, aggregated at the division level, such as Mining, Construction, Manufacturing, etc. We use all firms in Compustat dataset to calibrate the model for the representative firm in each industry and show those values in Table 1 (the calibration procedure is in the following subsection). The parameter values for the representative firms are largely consistent with the existent literature (e.g., Hennessy and Whited, 2005 and 2007; and DeAngelo, DeAngelo, and Whited, 2011).

[Insert Table 1 here]

For the comparative statics analysis, we assume the firm is at the outset of its life (i.e., $t = 0$) and the current state (K_0, L_0, B_0, z_0) is at the mean of the stationary distribution of profit shocks.[14]

[13] Ou (2011) and Vazza and Kraemer (2015) show that each credit rating has a certain default probability associated.

[14] The corresponding formulas for the current state are

$$z_0 = c^{\frac{1}{1-\rho}} e^{\frac{1}{2}\sigma^2 \frac{1}{(1-\rho^2)}}, \quad K_0 = z_0^{\frac{1}{1-(\alpha_K+\alpha_L)}} \Phi_1^*, \quad L_0 = z_0^{\frac{1}{1-(\alpha_K+\alpha_L)}} \Phi_2^*, \quad \text{and} \quad B_0 = \ell^* K_0. \tag{15}$$

The sensitivity analysis of the stock price with respect to model parameter values is in Table 2. Specifically, we show in the table the percentage change in share price when the parameter values increase by 1%. In our model, the parameters with the highest marginal effects are the elasticity of capital (α_K) and the persistence of profit shocks (ρ). A 1% increase in these parameter values augments the stock price between 7% and 12% across the different industries. As equation (8) suggests, both of these parameters directly increase the expected profits of the firm and, thus, have a substantial influence on firm size (i.e., and share price).[15]

Second in importance are constant c, the elasticity of labor (α_L), and the market cost of capital (r_A). The impact of these parameters on the stock price is between 1% and 4% (in absolute values) across the different industries. Equation (8) also shows that both c and α_L directly affect expected firm profitability, which explains their considerable impact on the stock price. The (negative) impact of r_A on share price is due to the strong effect of this parameter on the discounting of future cash flows, as shown in equation (7). The remaining parameters have a relatively small marginal effect of less than 1% (in absolute values).[16]

[Insert Table 2 here]

Overall, the results above suggest that the operating aspects of the firm play the most important role in the comparative statics analysis. Thus, firm primitives such as constant c, the persistence of profit shocks (ρ), the curvature of the production function with respect to capital (α_K) and labor (α_L), and the market cost of capital (r_A) should be the ones that receive the greatest attention in the calibration/estimation procedure previous to firm valuation. In addition, richer datasets than Compustat containing more accurate information about, for instance, gross profits and assets, could also improve the precision of the parameter estimates.

where Φ_1^*, Φ_2^*, and ℓ^* are as described in Proposition 1.

[15] Parameter α_K has received great attention in the literature of Industrial Organization; see, e.g., Ackerberg, Benkard, Berry, and Pakes, 2007, and the references therein.

[16] This analysis extends Lazzati and Menichini (2015a), who perform a sensitivity analysis of the stock price for a simpler model of the firm.

3.1 Calibration of Model Parameters

In this subsection, we describe how we calibrate the model parameters for the representative firm in each SIC industry. The parameters of interest are $c, \rho, \sigma, \alpha_K, \alpha_L, f, \delta, \omega, \tau, r_B, r_A, g$, and ξ. The sample includes all firms in Compustat database and covers the period 1950-2015.

Following Moyen (2004), we obtain parameters $c, \rho, \sigma, \alpha_K$ and α_L using the firm's autoregressive profit shock process of equation (1) and the gross profits function in equation (2). The data we use with these equations are Gross Profit (GP), Assets - Total (AT), and Number of Employees (EMP). We first log-linearize the gross profits function and obtain parameters α_K and α_L by doing an OLS regression.[17] We then use the residual term from that regression with the firm's autoregressive profit shock process to obtain parameters c, ρ, and σ.

We calibrate parameter f by averaging the ratio Selling, General, and Administrative Expense (XSGA)/Assets - Total (AT) for all firm-years in each industry. We follow the same procedure to get δ from the ratio of Depreciation and Amortization (DP) over Assets - Total (AT), ω from the fraction Staff Expense - Total (XLR)/Number of Employees (EMP), τ from the ratio Income Taxes - Total (TXT) over Pretax Income (PI), and r_B as the proportion Interest and Related Expense - Total (XINT)/Liabilities - Total (LT). We follow the procedure described by Kaplan and Ruback (1995) to derive r_A using CAPM after unlevering the equity beta.[18] We use the industry long-run growth rates in Jorgenson and Stiroh (2000) as parameter g for each SIC industry. Finally, we calibrate ξ using the fraction Liabilities - Total (LT)/Assets - Total (AT) in equation (14). We exclude firms with SIC 6000-6700 (i.e., Finance, Insurance, and Real Estate) as well as firms with SIC 9100-9900 (i.e., Public Administration). We also eliminate observations with missing data and trim those ratios at the lower and upper one-percentiles to reduce the effect of outliers and errors in the data. The final sample includes 62,036 firm-year observations.

[17] This is a standard procedure in the empirical economics literature (see, e.g., Balistreri, McDaniel, and Wong 2003; Fox and Smeets 2011; Young 2013).

[18] We use the market risk premiums published on Damodaran's website, and the 10-year T-Bond yields as the risk-free interest rate.

4 Testing the Performance of the Valuation Model

We finally study two fundamental aspects of our dynamic model. First, we analyze the consistency between the model estimates and market prices. That is, we address how close the predicted values by our model are to the actual market prices, on average. Complementing this analysis, we then examine how much of the observed variation in contemporaneous share prices is explained by the model estimates. Second, we investigate the possibility to use our model to predict future stock returns and, thus, exploit short-term differences between those two values.

To generate the following results, we value firms included in the S&P 100 Index in the period 1990-2015. We start describing the sample as well as the estimation procedure.

4.1 Estimation Procedure and Identification

Our sample contains the firms included in the S&P 100 Index in the period 1990-2015. Constituents of this index are large and mature U.S. companies and typically represent more than 40% of the market capitalization of the U.S. stock markets. We construct this sample using two data sources. Historical accounting data are obtained from the Compustat annual files, while the corresponding stock price data are obtained from the CRSP monthly files. We use the historical data for each firm during the period 1950-2015 to estimate its own parameters. In all our empirical analyses, we ensure that accounting data are known at the time the stock price is set in the exchange. Thus, we use the share price observed five months later than the fiscal-year-end of the firm. For instance, we match the accounting data of a December year-end firm with its closing stock price at the end of May of the following year. Because our objective is to do out-of-sample predictions, we estimate parameter values using the existing historical information up to the year prior to the valuation period.

The parameters we estimate for each firm are $c, \rho, \sigma, \alpha_K, \alpha_L, f, \delta, \omega, \tau$, r_B and ξ. The estimation technique we employ is the simulated method of moments (SMM). Succinctly, SMM attempts to find the model parameters that generate simulated moments as close as possible to the same moments computed with the data. We use the moments described in subsection (3.1) to do the estimation of model parameters. For each firm, identification in the empirical estimation comes from changes over time in the observed values of the data. The market cost of capital r_A for each firm is obtained following the procedure of Kaplan and Ruback (1995), as mentioned before. Finally, for each firm we use the corresponding industry long-run growth rate (g) reported by Jorgenson and Stiroh (2000).

To improve the accuracy of the estimation, we require firms to have at least 20 observations before the valuation period. We also eliminate observations with missing data and trim the ratios at the lower and upper one-percentiles to diminish the impact of outliers and errors in the data. The sample includes 2,548 firm-year observations.

4.2 Explanation of Contemporaneous Stock Prices

We start studying the consistency between our model estimates and market prices. To this end, for each firm-year observation in the sample, we construct the market-to-value ratio (P/V), which is the market value of equity (P) divided by the equity value estimated by our dynamic model (V). Panel A in Table 3 reports the summary statistics for this ratio, which we summarize next.

Result 1: The mean of P/V is around 1.02, which implies that the mean observation of the market value of equity is almost equal to the mean dynamic DDM estimate.

The previous result is important because it suggests that, in the long-run, our model produces estimates that are roughly equal to market prices. The median of P/V has a value of 0.95. We believe the difference between the mean and the median is reasonable because the market-to-value ratio is bounded below at zero but unbounded above, which creates a distribution of P/V that is skewed to the right. Panel B in Table 3 presents different measures of central tendency that further corroborate our first result.

[Insert Table 3 here]

Figure 1 shows the evolution of the mean value of the market-to-value ratio (line with crosses) over the period 1990-2013. It shows that the ratio is close to the value of 1 during each sub-period along the whole observation window. The figure also shows that the ratio moves away from 1 during periods of strong market movements, such as around the culmination of the stock market surge at the end of the 1990s, returning toward 1 in the subsequent periods.

[Insert Figure 1 here]

The previous analyses showed that the unconditional means of market values and model estimates are very close to each other. We now study the level of linear association between those two variables as well as the proportion of the variation in current prices that is explained by the predictions of our model. This analysis highlights the goodness of fit of our valuation model. Accordingly, we estimate the following basic regression

$$P_{i,t} = \alpha + \beta V_{i,t} + \epsilon_{i,t} \tag{16}$$

where P denotes the market value of equity and V represents the value estimated by the model. In this specification, i indexes firms, t indexes time periods, and $\epsilon_{i,t}$ is an *iid* random term. In theory, an intercept of zero and a slope of one would suggest that our model produces unbiased estimates of market values in each period. The results in the first column in Table 4 confirm this possibility. We summarize next the main findings.[19]

Result 2: We cannot reject the null hypothesis that $\alpha = 0$ even at the 10% level. In addition, as expected, the estimated slope is very close to one $\left(\widehat{\beta} = 1.03\right)$. With an adjusted r-squared of 83.1%, the dynamic DDM explains a large proportion of the variation in current stock prices.

The r-squared generated by our model is larger than the ones found by related valuation studies, such as Bernard (1995), Frankel and Lee (1998), and Spiegel and Tookes (2013) —their r-squared coefficients are 68%, 67%, and 43%, respectively.

[19] As benchmark, the second column in Table 5 displays the results from regressing the market value of equity on the book value of equity. As usual, the intercept is significantly different from zero. In addition, the slope $\left(\widehat{\beta} = 2.16\right)$ is statistically significantly different from one. Consistently with previous studies, the adjusted r-squared is 56.4%. Finally, the third column shows the results from regressing the market value of equity on net earnings. The intercept turns out to be significantly different from zero and the slope $\left(\widehat{\beta} = 4.60\right)$ is statistically significantly different from one. The adjusted r-squared is only 22.1%

[Insert Table 4 here]

Overall, the results in this subsection suggest that our dynamic DDM produces equity value estimates that are consistent with market prices and explain a large part of the variation in current stock prices. We next explore the possibility to use the model to economically exploit short-run differences between market values and model estimates.

4.3 Forecast of Future Stock Returns

In the previous subsection, we showed that our dynamic DDM produces value estimates that are, on average, very close to contemporaneous share prices. Nevertheless, the linear association is not perfect, which also means that there are temporary or short-run deviations between market prices (P) and estimated values (V) for individual stocks. We next show that these temporary differences can be economically exploited.

To achieve our goal, we first construct portfolios based on the ranking of (demeaned) P/V of the firms in the sample.[20] We then form quintile portfolios where lower quintiles include firms with low P/V and higher quintiles include firms with high P/V. Thus, firms in the lower quintile portfolios have market prices that are low relative to our model predictions and might experience higher stock returns in the near future than firms in the higher quintile portfolios. The opposite reasoning holds for firms in the higher quintile portfolios. The last step consists in implementing a simple portfolio strategy by taking a long position in the bottom quintile portfolio and a short position in the top quintile portfolio.

To evaluate this strategy, we form quintile portfolios from 1990 through 2009 (i.e., 20 periods) and track the cumulative returns of the strategy over the following 36 months. Panel A in Table 5 displays the outcomes of this strategy and we highlight them in the following result.

Result 3: The column labeled Q1-Q5 in Table 5 shows that the strategy earns 20.57%, 36.31%, and 51.69% on average over the 12, 24, and 36 months following portfolio formation, respectively. These returns are statistically significant.

[20]We demean P/V of each firm to eliminate the effect of systematic differences between the market and the estimated value of equity.

In the eighth column of Table 6 we exhibit the same returns, but adjusted by the risk implied by the strategy (i.e., the ratio return/standard deviation of returns). The last column reports the percentage of periods in which the strategy earned positive returns. Specifically, it shows that, after 12 and 24 months of portfolio formation, the strategy obtained positive results in 95% of the periods (i.e., 19 out of the 20 years), and that after 36 months of portfolio formation the strategy produced positive returns in 100% of the periods (i.e., 20 out of the 20 years). Overall, these results provide evidence that differences between market prices (P) and our estimated values (V) can indeed be profitably exploited.

To appreciate the magnitude of our previous results, we compare them with portfolio strategies based on the market-to-book ratio (P/B) and the price-earnings ratio (P/E). These two are among the most well-known accounting-based ratios that exhibit predictive power for stock returns (Fama and French, 1992) and have been used before to assess valuation outcomes (e.g., Frankel and Lee, 1998). The following result summarizes our findings.

Result 4: Panels B and C of Table 5 show that the portfolio strategy using P/V considerably outperforms the P/B and P/E strategies in each of the three investment horizons. For example, over the period of 36 months, the P/V portfolios yield roughly 21% more than the P/B portfolios (51.69% versus 30.00%) and approximately 31% more than the P/E portfolios (51.69% versus 19.98%), on average.

Column 8 shows that this result still holds if we adjust the corresponding returns in terms of risk.[21] Finally, the percentage of winner periods with the strategy based on our model estimates is larger than those with the P/B and P/E portfolios.

[Insert Table 5 here]

To highlight our previous results, Figure 2 graphically displays the evolution of the average returns of the P/V, P/B, and P/E portfolios over the 36 months after portfolio formation.

[21] To evaluate the robustness of this outcome, we also computed two other common measures of return-to-variability —the Sharpe and Treynor ratios. We obtained analogue results.

Overlaying the three lines are fitted curves that display the general trends, and corroborate our previous observations. The concavity or flattening of the general trends reflect the fact that the benefits from the information available at the moment of portfolio formation naturally diminish as time passes.

[Insert Figure 2 here]

We finally relate our results to the Fama/French 3-factor model of stock returns (Fama and French, 1993). Our objective is to analyze whether the positive returns of the P/V portfolios shown in Table 5 can be explained by the exposure to the risk factors proposed by that model. The Fama/French 3-factor model suggests that the expected return on a stock in excess of the risk-free rate, $E(R) - R_f$, can be explained by the next three factors.

1. The expected return on the market portfolio in excess of the risk-free rate, $E(R_M) - R_f$. This factor proxies for systematic risk.

2. The expected return on a portfolio of small stocks minus the expected return on a portfolio of big stocks (SMB, Small Minus Big). As Fama and French (1993) suggests, this variable might be associated with a common risk factor that explains the observed negative relation between firm size and average return.

3. The expected return on a portfolio of value stocks (high book-to-market ratio stocks) minus the expected return on a portfolio of growth stocks (low book-to-market ratio stocks) (HML, High Minus Low.) The same authors suggest that this variable might be associated with a common risk factor that explains the observed positive relation between the book-to-market ratio and average return.

Following the specification proposed by Fama and French (1993), we perform the next regression

$$R_{i,t} - R_f = \alpha + \beta_1 (R_M - R_f) + \beta_2 SMB + \beta_3 HML + \epsilon_{i,t} \tag{17}$$

where we regress the excess (monthly) returns of the P/V portfolios on the three factors.[22] In this equation, i indexes portfolios, t indexes time periods, and $\epsilon_{i,t}$ is an *iid* random term. We next summarize the results displayed in Table 6.

Result 5: We cannot reject the null hypothesis that the three factor loadings are zero even at the 10% level. On the contrary, the intercept, α, has a positive and significant value of 0.009 (i.e., 0.9% per month). These results suggest that the P/V portfolios generate returns that are significantly positive and uncorrelated with those risk factors. The low r-squared of 0.0004 confirms this finding.

In other words, the positive returns obtained by the P/V portfolios do not seem to be explained by exposure to the risk factors in the Fama/French 3-factor model.

[Insert Table 6 here]

5 Conclusion

We derive a dynamic version of the dividend discount model (DDM) in closed-form and evaluate its empirical performance. The implementation of our method relies on widely available financial data and implies a new valuation approach that involves two simple steps. First, model parameters (i.e., the proxies for the economic fundamentals of the firm) must be calibrated or estimated. Second, the model uses current data on book value of equity and gross profits to determine the stock price. It does so by systematically projecting the infinite sequence of future expected dividends. Thus, our model does not require to actually forecast any future value (including a terminal value), helping to reduce the degree of discretion on the user side.

The empirical evaluation of the dynamic DDM yields promising results. First, we find that our model forecasts stock prices consistently, that is, model estimates are very close to market prices on average. Second, the model explains a large proportion (around 83%) of the observed variability in current stock prices. Finally, we find that the model can be used to predict future

[22] We use the data on the three factors published on French's website.

stock returns in the cross-section of firms by exploiting temporary differences between market prices and model estimates. For instance, constructing portfolios based on the ratio of the market value of equity to the equity value estimated by the model, we find that simple portfolio strategies earn considerably positive returns over the three following years (e.g., an average of around 21%, 36%, and 52% returns after 1, 2, and 3 years, respectively, of portfolio formation).

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6 Appendix 1: Proofs

The proof of Proposition 1 requires an intermediate result that we present next.

Lemma 2 *The maximum level of book leverage in each period is given by*

$$\ell^* = \frac{1 + \left[e^{\sigma\left(x_c' - \frac{1}{2}\sigma\right)} \Phi_1^{*\alpha_K - 1} \Phi_2^{*\alpha_L} - f - \delta - \omega \frac{\Phi_2^*}{\Phi_1^*} \right] (1 - \tau) - \xi}{1 + r_B (1 - \tau)}. \tag{18}$$

Proof Given the value of x_c' (i.e., for an arbitrary probability of bankruptcy $\lambda(x_c')$), the firm goes into bankruptcy when

$$\left(z_c' K'^{\alpha_K} L'^{\alpha_L} - fK' - \delta K' - \omega L' - r_B \ell K' \right) (1 - \tau) + K' - \ell K' - \xi K' < 0 \tag{19}$$

where $z_c' = cz^{\rho} e^{\sigma x_c'}$ is the cutoff value of z such that the probability of $z' < z_c'$ is $\lambda(x_c')$. The maximum book leverage ratio consistent with probability of bankruptcy $\lambda(x_c')$ then satisfies

$$\left(z_c' K'^{\alpha_K} L'^{\alpha_L} - fK' - \delta K' - \omega L' - r_B \ell^* K' \right) (1 - \tau) + K' - \ell^* K' - \xi K' = 0. \tag{20}$$

Working on the previous expression (and using the optimal policies derived in equation (32)), we can obtain the maximum level of book leverage as

$$\ell^* = \frac{1 + \left[e^{\sigma\left(x_c' - \frac{1}{2}\sigma\right)} \Phi_1^{*\alpha_K - 1} \Phi_2^{*\alpha_L} - f - \delta - \omega \frac{\Phi_2^*}{\Phi_1^*} \right] (1 - \tau) - \xi}{1 + r_B (1 - \tau)} \tag{21}$$

which completes the proof.[23] ■

Proof of Proposition 1

The market value of equity can be expressed as

$$S_0(K_0, L_0, B_0, z_0) = \max_{\{K_{t+1}, L_{t+1}, B_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \frac{(1+g)^t}{\prod_{j=0}^{t} \left(1 + r_{S_j}\right)} Y_t. \tag{22}$$

Because we use the Adjusted Present Value method of firm valuation, we solve the problem of the firm in equation (22) in three steps. First, we determine the value of the unlevered firm, $S_{u_0}(K_0, L_0, z_0)$. Second, we solve for optimal debt and compute the present value of the financing side effects. Finally, we obtain the value of the levered firm in equation (22).

[23] The restriction $\left(f + \delta + \omega \frac{\Phi_2^*}{\Phi_1^*} \right) (1 - \tau) + \xi \leq 1$ described in Section 2 also guarantees that $\ell^* \geq 0$.

The market value of equity for the unlevered firm can be expressed as

$$S_{u_0}\left(K_0, L_0, z_0\right) = \max_{\{K_{t+1}, L_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \left(\frac{1+g}{1+r_A}\right)^t Y_{u_t} \tag{23}$$

where $Y_{u_t} = N_{u_t} - (K_{t+1} - K_t)$ and $N_{u_t} = \left(z_t K_t^{\alpha_K} L_t^{\alpha_L} - f K_t - \delta K_t - \omega L_t\right)(1-\tau)$. We let normalized variables with primes indicate values in the next period and normalized variables with no primes denote current values. Then, the Bellman equation for the problem of the firm in equation (23) is given by

$$\begin{array}{l} S_u\left(K, L, z\right) = \max_{K', L'} \{\left(z K^{\alpha_K} L^{\alpha_L} - f K - \delta K - \omega L\right)(1-\tau) - (1+g) K' + K + \\ \frac{(1+g)}{(1+r_A)} E\left[S_u\left(K', L', z'\right)|z\right]\Big\}. \end{array} . \tag{24}$$

We use the guess and verify method as the proof strategy. Thus, we start by guessing that the solution is given by

$$S_u\left(K, L, z\right) = \left(z K^{\alpha_K} L^{\alpha_L} - f K - \delta K - \omega L\right)(1-\tau) + K + M(z) P_u^* \tag{25}$$

where

$$M(z) = e^{-\frac{1}{2}\sigma^2 \frac{(\alpha_K+\alpha_L)}{[1-(\alpha_K+\alpha_L)]^2}} \sum_{n=1}^{\infty} \left\{ \left(\frac{1+g}{1+r_A}\right)^n \left(c^{\frac{1-\rho^n}{1-\rho}} z^{\rho^n} e^{\frac{1}{2}\sigma^2 \frac{\left(1-\rho^{2n}\right)}{(1-\rho^2)} \frac{1}{[1-(\alpha_K+\alpha_L)]}} \right)^{\frac{1}{1-(\alpha_K+\alpha_L)}} \right\}, \tag{26}$$

$$P_u^* = \left(\Phi_1^{*\alpha_K} \Phi_2^{*\alpha_L} - f\Phi_1^* - \delta\Phi_1^* - \omega\Phi_2^*\right)(1-\tau) - r_A \Phi_1^*, \tag{27}$$

$$\Phi_1^* = \left[\left(\frac{\alpha_K}{\frac{r_A}{1-\tau} + f + \delta}\right)^{1-\alpha_L} \left(\frac{\alpha_L}{\omega}\right)^{\alpha_L}\right]^{\frac{1}{1-(\alpha_K+\alpha_L)}}, \text{ and} \tag{28}$$

$$\Phi_2^* = \left[\left(\frac{\alpha_K}{\frac{r_A}{1-\tau} + f + \delta}\right)^{\alpha_K} \left(\frac{\alpha_L}{\omega}\right)^{1-\alpha_K}\right]^{\frac{1}{1-(\alpha_K+\alpha_L)}}. \tag{29}$$

We obtained this initial guess as the solution of equation (24) by the backward induction method.

We now verify our guess. To this end, let us write

$$S_u\left(K, L, z\right) = \max_{K', L'} \left\{F\left(K', L', K, L, z\right)\right\} \tag{30}$$

with F defined as the objective function in equation (24).

The FOC for this problem is

$$\begin{aligned}\partial F\left(K', L', K, L, z\right)/\partial K' &= -\left(1+g\right) + \tfrac{(1+g)}{(1+r_A)}\left[\left(E\left[z'|z\right]\alpha_K K^{*\alpha_K-1} L^{*\alpha_L} - f - \delta\right)\left(1-\tau\right)+1\right] = 0 \\ \partial F\left(K', L', K, L, z\right)/\partial L' &= \tfrac{(1+g)}{(1+r_A)}\left(E\left[z'|z\right] K^{*\alpha_K}\alpha_L L^{*\alpha_L-1} - \omega\right)\left(1-\tau\right) = 0\end{aligned} \tag{31}$$

and optimal capital and labor turn out to be

$$K^* = E\left[z'|z\right]^{\frac{1}{1-(\alpha_K+\alpha_L)}}\Phi_1^* \quad \text{and} \quad L^* = E\left[z'|z\right]^{\frac{1}{1-(\alpha_K+\alpha_L)}}\Phi_2^* \tag{32}$$

where Φ_1^* and Φ_2^* are as in equations (28) and (29), respectively.

Finally, the market value of equity for the unlevered firm becomes

$$\begin{aligned}S_u\left(K, L, z\right) &= \left(zK^{\alpha_K}L^{\alpha_L} - fK - \delta K - \omega L\right)\left(1-\tau\right) - \left(1+g\right)K^* + K + \\ &\quad \tfrac{(1+g)}{(1+r_A)}\left[\left(E\left[z'|z\right]K^{*\alpha_K}L^{*\alpha_L} - fK^* - \delta K^* - \omega L^*\right)\left(1-\tau\right) + K^* + \right. \\ &\quad \left. E\left[M\left(z'\right)|z\right]P_u^*\right] \\ &= \left(zK^{\alpha_K}L^{\alpha_L} - fK - \delta K - \omega L\right)\left(1-\tau\right) + K - \left(1+g\right)E\left[z'|z\right]^{\frac{1}{1-(\alpha_K+\alpha_L)}}\Phi_1^* + \\ &\quad \tfrac{(1+g)}{(1+r_A)}\left\{E\left[z'|z\right]^{\frac{1}{1-(\alpha_K+\alpha_L)}}\left[\left(\Phi_1^{*\alpha_K}\Phi_2^{*\alpha_L} - f\Phi_1^* - \delta\Phi_1^* - \omega\Phi_2^*\right)\left(1-\tau\right) + \Phi_1^*\right] + \right. \\ &\quad \left. E\left[M\left(z'\right)|z\right]P_u^*\right\} \\ &= \left(zK^{\alpha_K}L^{\alpha_L} - fK - \delta K - \omega L\right)\left(1-\tau\right) + K + \\ &\quad \tfrac{(1+g)}{(1+r_A)}\left(e^{-\frac{1}{2}\sigma^2\frac{(\alpha_K+\alpha_L)}{[1-(\alpha_K+\alpha_L)]^2}}E\left[z'^{\frac{1}{1-(\alpha_K+\alpha_L)_K}}|z\right] + E\left[M\left(z'\right)|z\right]\right)P_u^* \\ &= \left(zK^{\alpha_K}L^{\alpha_L} - fK - \delta K - \omega L\right)\left(1-\tau\right) + K + M\left(z\right)P_u^*\end{aligned} \tag{33}$$

which is equivalent to our initial guess in equation (25).

Next, we obtain optimal risky debt. In each period, the firm solves the following problem

$$\max_{B', x_c'}\left\{B' - \frac{1}{(1+r_B)}\left\{B'\left[1 + r_B\left(1-\tau\right)\right] - \lambda\left(x_c'\right)\xi K'\right\}\right\}. \tag{34}$$

where x_c' is the cutoff value of the standard normal random variable x such that the probability of $x' < x_c'$ is $\lambda\left(x_c'\right)$.

The above problem can be solved in 2 steps. In the first step, given the value of x_c' (i.e., for an arbitrary probability of bankruptcy $\lambda\left(x_c'\right)$), the firm chooses optimal debt B^*. Defining F as the objective function in equation (34), the FOC turns out to be $\partial F\left(K', B', x_c'\right)/\partial B' = \frac{1}{(1+r_B)}r_B\tau >$

0. Because $r_B > 0$ and $\tau > 0$, the firm increases debt as much as possible to maximize the tax benefits of debt. Then, optimal debt is $B^* = \ell^* K^*$ where

$$\ell^* = \frac{1 + \left[e^{\sigma\left(x_c' - \frac{1}{2}\sigma\right)} \Phi_1^{*\alpha_K - 1} \Phi_2^{*\alpha_L} - f - \delta - \omega \frac{\Phi_2^*}{\Phi_1^*} \right] (1-\tau) - \xi}{1 + r_B (1-\tau)}, \tag{35}$$

as shown in Lemma 2. In the second step, the firm selects the value of x_c' that maximizes the present value of the financing side effects. Accordingly, the problem of the firm becomes

$$\max_{x_c'} \left\{ \ell^* \left(x_c'\right) K' - \frac{1}{(1+r_B)} \left\{ \ell^* \left(x_c'\right) K' \left[1 + r_B (1-\tau)\right] - \lambda \left(x_c'\right) \xi K' \right\} \right\}. \tag{36}$$

After solving the previous problem, the optimal probability of bankruptcy is given by

$$\lambda^* = \int_{-\infty}^{x_c^*} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \tag{37}$$

where

$$x_c^* = -\sigma - \sqrt{2 \left\{ \sigma^2 + \ln \left\{ \frac{\left[1 + \frac{1}{r_B(1-\tau)}\right] \xi}{\sqrt{2\pi} \tau \sigma \Phi_1^{*\alpha_K - 1} \Phi_2^{*\alpha_L}} \right\} \right\}}. \tag{38}$$

Finally, the present value of the financing side effects turns out to be

$$\begin{aligned} Q(z) &= \left(\tfrac{1+g}{1+r_A}\right) \left\{ \left(\tfrac{1+r_A}{1+r_B}\right) \left(r_B \tau \ell^* - \lambda^* \xi\right) K^* + E\left[Q(z') \,|z\right] \right\} \\ &= M(z) \left(\tfrac{1+r_A}{1+r_B}\right) \left(r_B \tau \ell^* - \lambda^* \xi\right) \Phi_1^* \end{aligned} \tag{39}$$

where $M(z)$ is as in equation (26). Under this financial policy, the amount of debt and interest payments will vary with the future asset cash flows (i.e., they depend on future firm performance). Then, because the financing side effects will have a level of risk in line with that of the firm cash flows, we use the cost of capital, r_A, as the discount rate. This feature of the model is consistent with Kaplan and Ruback (1995).

The third step consists in obtaining the market value of equity for the levered firm that does not go into bankruptcy. If we assume the firm used debt B in the previous period, and now has to pay interest $r_B B (1-\tau)$, then the stock price for the levered firm is

$$\begin{aligned} S(K, L, B, z) &= S_u(K, L, z) + M(z) \left(\tfrac{1+r_A}{1+r_B}\right) \left(r_B \tau \ell^* - \lambda^* \xi\right) \Phi_1^* - B - r_B B (1-\tau) \\ &= \left(z K^{\alpha_K} L^{\alpha_L} - fK - \delta K - \omega L - r_B B\right)(1-\tau) + K - B + G(z) \end{aligned} \tag{40}$$

where $G(z) = M(z) P^*$ and variable P^* takes the form

$$P^* = \left(\Phi_1^{*\alpha_K} \Phi_2^{*\alpha_L} - f\Phi_1^* - \delta\Phi_1^* - \omega\Phi_2^*\right)(1-\tau) - r_A\Phi_1^* + \left(\frac{1+r_A}{1+r_B}\right)(r_B\tau\ell^* - \lambda^*\xi)\,\Phi_1^*. \tag{41}$$

Finally, the optimal decisions of the firm are given by

$$\begin{aligned} K^*(z_t) &= (1+g)\, E\left[z_{t+1}|z_t\right]^{\frac{1}{1-(\alpha_K+\alpha_L)}} \Phi_1^*, \\ L^*(z_t) &= (1+g)\, E\left[z_{t+1}|z_t\right]^{\frac{1}{1-(\alpha_K+\alpha_L)}} \Phi_2^*, \text{ and} \\ B^*(z_t) &= \ell^* K^*(z_t) \end{aligned} \tag{42}$$

and the market value of equity is

$$\begin{aligned} S(K_t, L_t, B_t, z_t) = {} & \left[(1+g)^{t[1-(\alpha_K+\alpha_L)]} z_t K_t^{\alpha_K} L_t^{\alpha_L} - fK_t - \delta K_t - \omega L_t - r_B B_t\right](1-\tau) + \\ & K_t - B_t + G(z_t) \end{aligned} \tag{43}$$

as shown in Proposition 1. ■

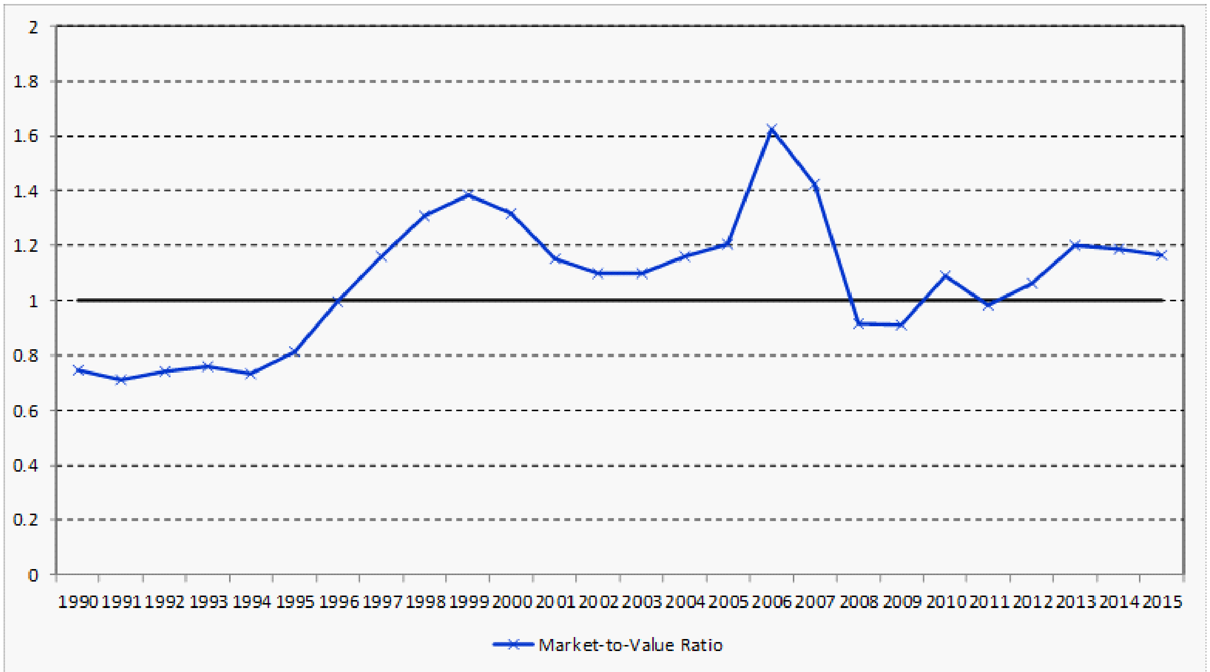

Figure 1. Evolution of the market-to-value ratio. The figure displays the evolution over time of the mean market-to-value ratio for a sample of firms included in the S&P 100 Index in the period 1990-2015. The market-to-value ratio is the market value of equity divided by the value estimated by the model.

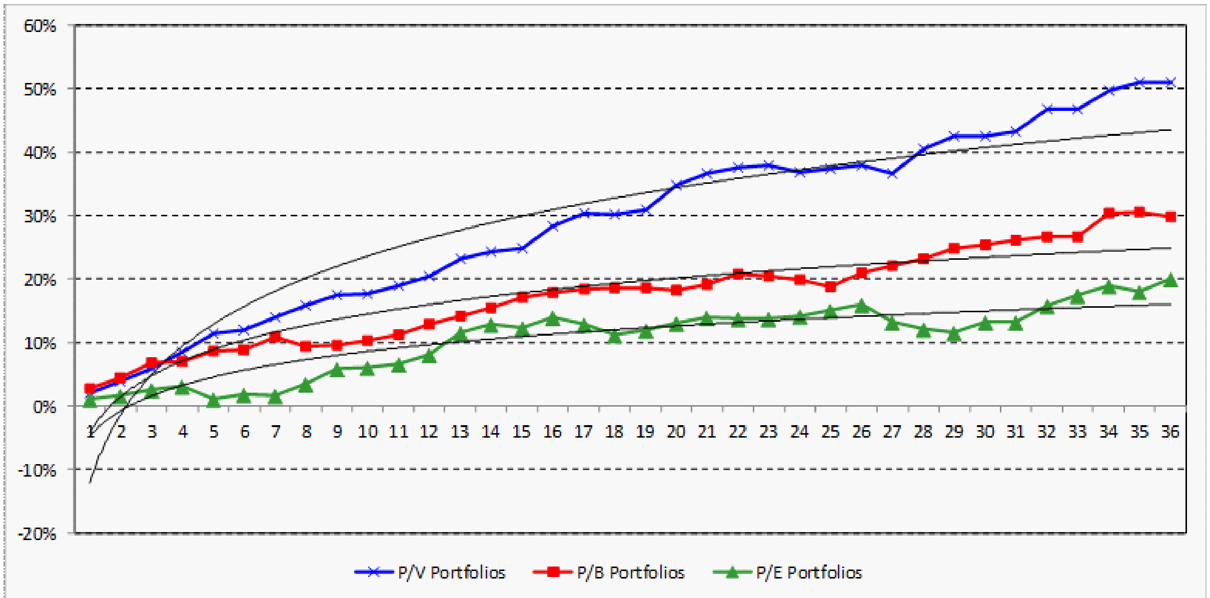

Figure 2. Cumulative strategy returns. The figure displays the cumulative returns of the portfolio strategies for the P/V, P/B, and P/E portfolios during the 36 months following portfolio formation. These portfolios are constructed with a sample of firms included in the S&P 100 Index in the period 1990-2015. P/V is the market-to-value ratio (i.e., the market value of equity divided by the equity value estimated by the model). P/B is the market-to-book ratio (i.e., the market value of equity divided by the book value of equity). P/E is the price-earnings ratio (i.e., the market value of equity divided by the firm's net income). Portfolios are formed by sorting firms into quintiles according to their P/V, P/B, and P/E ratios at the end of May of each year. The portfolio strategy consists in buying firms in the bottom quintile and selling firms in the top quintile.

Table 1

Cross-Sectional Parameter Values

The table presents the values used to parameterize the dynamic model of the firm for the different SIC industries. The parameters are constant c, the persistence of profit shocks (ρ), the standard deviation of the innovation term (σ), the concavity of the production function with respect to capital (α_K) and labor (α_L), the operating costs (f), the capital depreciation rate (δ), labor wages (ω), the corporate income tax rate (τ), the market cost of debt (r_B), the market cost of capital (r_A), the growth rate (g), and the bankruptcy costs (ξ).

Industry	c	ρ	σ	α_K	α_L	f	δ	ω	τ	r_B	r_A	g	ξ
Agriculture, Forestry, And Fishing	101.379	0.400	0.302	0.354	0.119	0.528	0.042	0.283	0.296	0.052	0.086	0.017	0.235
Mining	110.289	0.373	0.392	0.322	0.139	0.392	0.081	0.391	0.241	0.053	0.074	0.013	0.340
Construction	120.923	0.382	0.380	0.373	0.092	0.670	0.036	0.328	0.325	0.049	0.094	0.014	0.301
Manufacturing	156.190	0.440	0.336	0.333	0.131	0.602	0.029	0.331	0.340	0.039	0.088	0.026	0.422
Transportation and Public Utilities	78.874	0.453	0.229	0.360	0.143	0.451	0.062	0.337	0.344	0.044	0.078	0.028	0.357
Wholesale Trade	107.833	0.373	0.221	0.400	0.082	0.629	0.048	0.377	0.332	0.045	0.091	0.037	0.395
Retail Trade	115.374	0.408	0.200	0.386	0.092	0.507	0.045	0.401	0.351	0.042	0.085	0.032	0.454
Services	178.002	0.370	0.272	0.395	0.108	0.627	0.070	0.411	0.330	0.044	0.103	0.039	0.570
Average	121.108	0.400	0.292	0.366	0.113	0.551	0.052	0.357	0.320	0.046	0.088	0.026	0.384

Table 2

Comparative Statics Analysis of Market Equity Value

The table exhibits the results of the sensitivity analysis of the stock price for different SIC industries. The columns show the percentage variation in share price when the corresponding parameter changes by 1%. The parameters are constant c, the persistence of profit shocks (ρ), the standard deviation of the innovation term (σ), the concavity of the production function with respect to capital (α_K) and labor (α_L), the operating costs (f), the capital depreciation rate (δ), labor wages (ω), the corporate income tax rate (τ), the market cost of debt (r_B), the market cost of capital (r_A), the growth rate (g), and the bankruptcy costs (ξ).

Industry	c	ρ	σ	α_K	α_L	f	δ	ω	τ	r_B	r_A	g	ξ
Agriculture, Forestry, And Fishing	2.697	8.485	0.197	8.935	2.613	-0.463	-0.040	-0.215	-0.434	0.008	-1.207	0.234	-0.009
Mining	2.582	7.399	0.325	7.722	2.944	-0.378	-0.083	-0.247	-0.325	0.006	-1.181	0.205	-0.006
Construction	2.541	7.722	0.298	9.144	1.921	-0.495	-0.029	-0.162	-0.495	0.008	-1.131	0.170	-0.006
Manufacturing	2.882	11.733	0.263	9.700	3.432	-0.449	-0.023	-0.235	-0.539	0.006	-1.369	0.399	-0.007
Transportation and Public Utilities	3.226	11.939	0.135	10.842	3.879	-0.482	-0.070	-0.274	-0.548	0.015	-1.537	0.539	-0.009
Wholesale Trade	2.722	7.706	0.103	10.335	1.764	-0.553	-0.045	-0.151	-0.522	0.013	-1.629	0.647	-0.009
Retail Trade	2.839	9.449	0.086	10.512	2.134	-0.508	-0.047	-0.167	-0.570	0.014	-1.585	0.591	-0.011
Services	2.771	8.612	0.165	12.116	2.850	-0.532	-0.063	-0.207	-0.528	0.006	-1.562	0.584	-0.012
Average	2.782	9.131	0.197	9.913	2.692	-0.483	-0.050	-0.207	-0.495	0.009	-1.400	0.421	-0.009

Table 3

Valuation Results

The table shows the valuation results of the dynamic DDM for a sample of firms included in the S&P 100 Index in the period 1990-2015. P/V is the market-to-value ratio (i.e., the market value of equity divided by the equity value estimated by the model). The $\log(P/V)$ results derive from computing the log of the market-to-value ratio. The first line in Panel B shows the percentage of times that the value estimated by the model is within 15% of the market value of equity. The second line in Panel B shows the median value of the absolute difference between the equity value estimated by the model and the market value of equity (in percent). The third line in Panel B shows the median value of the squared difference between the value estimated by the model and the market value of equity (in percent). Standard errors are in parentheses.

	P/V
Panel A: Summary Statistics	
Mean	1.02
	(0.58)
Median	0.95
Interquartile Range	0.56
Panel B: Performance Measures	
Percentage within 15%	26.78%
Median Absolute Error	29.33%
Median Squared Error	8.60%

Table 4

Regression of the Market Value of Equity

The table shows the results from different cross-sectional regressions of the market value of equity. In column (1), the regressor is the equity value estimated by the model; in column (2), the regressor is the book value of equity; and in column (3), the regressor is net income. The sample is composed of firms included in the S&P 100 Index in the period 1990-2015. Standard errors are in parentheses.

Variable	*(1)*	*(2)*	*(3)*
Intercept	-1243.62	16989.23***	38654.78***
	(1141.88)	(1630.93)	(1840.25)
Equity Value Estimate	1.03**		
	(0.01)		
Book Value of Equity		2.16***	
		(0.06)	
Net Earnings			4.6***
			(0.31)
Adjusted R^2	0.831	0.564	0.221

Table 5

Cumulative Portfolio Returns

The table presents the cumulative portfolio returns of three different strategies. *P/V* Portfolios is the strategy that constructs portfolios based on the ranking of the market-to-value ratio (*P/V*); *P/B* Portfolios is the strategy that builds portfolios based on the ranking of the market-to-book ratio (*P/B*); and *P/E* Portfolios is the strategy that constructs portfolios based on the ranking of the price-earnings ratio (*P/E*). *Ret12*, *Ret24*, and *Ret36* are the average 12-month, 24-month, and 36-month portfolio returns of each strategy, respectively. *Q1-Q5* is the average spread of returns between the lowest (*Q1*) and highest (*Q5*) quintile portfolios. *Ret/Risk* is the ratio of the portfolio return to its standard deviation of returns. *% Winners* is the percentage of periods (out of 20 years) in which the strategy yielded positive returns.

	Q1	*Q2*	*Q3*	*Q4*	*Q5*	*Q1-Q5*	*Std Error*	*Ret/Risk*	*% Winners*
					Panel A: P/V Portfolios				
Ret12	16.56%	13.23%	4.32%	2.99%	-4.01%	20.57%***	0.15	1.37	95.00%
Ret24	30.12%	20.20%	8.98%	4.30%	-6.19%	36.31%***	0.22	1.65	95.00%
Ret36	40.40%	29.43%	12.89%	7.82%	-11.29%	51.69%***	0.47	1.10	100.00%
					Panel B: P/B Portfolios				
Ret12	11.38%	7.88%	5.66%	3.98%	-1.89%	13.27%***	0.20	0.66	80.00%
Ret24	20.87%	17.90%	9.45%	4.43%	1.53%	19.34%***	0.32	0.60	85.00%
Ret36	35.26%	24.68%	12.34%	8.78%	5.26%	30.00%***	0.40	0.75	85.00%
					Panel C: P/E Portfolios				
Ret12	10.73%	7.31%	4.68%	3.58%	2.68%	8.05%**	0.14	0.59	70.00%
Ret24	19.66%	12.60%	9.59%	7.81%	5.52%	14.14%***	0.21	0.69	80.00%
Ret36	30.57%	18.53%	12.17%	10.96%	10.59%	19.98%**	0.32	0.62	75.00%

Table 6

Risk Exposure of the Portfolios

The table shows the results from regressions of the quintile portfolios' excess returns on the 3 Fama/French risk factors. The regressor is the excess return of the P/V Portfolios, which is the strategy that constructs portfolios based on the ranking of the market-to-value ratio (P/V). *Rm-Rf* is the excess return on the market, *SMB* (Small Minus Big) is the return on a portfolio of small stocks minus the return on a portfolio of big stocks, and *HML* (High Minus Low) is the return on a portfolio of value stocks minus the return on a portfolio of growth stocks. The sample is composed of the portfolio returns during the period 1990-2015. Standard errors are in parentheses.

Variable	*P/V Portfolios*
Intercept	0.009***
	(0.002)
Rm-Rf	0.016
	(0.040)
SMB	-0.026
	(0.056)
HML	0.004
	(0.042)
R^2	0.0004