



Center for Analytical Finance
University of California, Santa Cruz

Working Paper No. 14

**Dynamic Model of Firm Valuation:
A New Methodology and its Empirical Validity**

June 25, 2015

Natalia Lazzati and Amilcar A. Menichini*

Abstract

This study derives a dynamic version of the dividend discount model and assesses its empirical validity. The valuation method we propose can be easily implemented and uses widely available financial data. We find that our model produces equity value estimates that are, on average, very close to market prices, and explains a large proportion of the variation observed in contemporaneous share prices. However, we also find temporary deviations between the stock prices and model estimates that can be economically exploited by a simple buy-and-hold portfolio strategy. The strategy we implement earns on average 22%, 37%, and 49% returns after one, two, and three years of portfolio formation, respectively.

JEL classification: G31, G32

Keywords: Firm Valuation; Dividend Discount Model; Gordon Growth Model; Dynamic Programming

* Natalia Lazzati is at the Department of Economics, University of California, Santa Cruz, CA 95064. Amilcar A. Menichini is at the Graduate School of Business and Public Policy, Naval Postgraduate School, Monterey, CA 93943 (e-mail: aamenich@nps.edu).

Dynamic Model of Firm Valuation: A New Methodology and its Empirical Validity

Natalia Lazzati and Amilcar A. Menichini*

June 25, 2015

Abstract

This study derives a dynamic version of the dividend discount model and assesses its empirical validity. The valuation method we propose can be easily implemented and uses widely available financial data. We find that our model produces equity value estimates that are, on average, very close to market prices, and explains a large proportion of the variation observed in contemporaneous share prices. However, we also find temporary deviations between the stock prices and model estimates that can be economically exploited by a simple buy-and-hold portfolio strategy. The strategy we implement earns on average 22%, 37%, and 49% returns after one, two, and three years of portfolio formation, respectively.

JEL classification: G31, G32

Keywords: Firm Valuation; Dividend Discount Model; Gordon Growth Model; Dynamic Programming

*Natalia Lazzati is from the Department of Economics, University of Michigan, Ann Arbor, MI 48109 (e-mail: nlazzati@umich.edu). Amilcar A. Menichini is from the Graduate School of Business and Public Policy, Naval Postgraduate School, Monterey, CA 93943 (e-mail: aamenich@nps.edu).

1 Introduction

We derive a dynamic model of the firm in closed-form that can be used for actual firm valuation. To test its empirical validity, we price firms currently in the S&P 100 Index and evaluate the results from different perspectives. First, we find that the model produces consistent forecasts of stock prices. That is, we compute the ratio of the actual market prices to the values predicted by our model and its median (mean) value turns out to be 1.01 (1.12). Second, we investigate the proportion of the variation in current stock prices that our model estimates can explain. We find that our value estimates explain around 75% of the variation in current market prices. However, we also find that there are temporary or short-run deviations between market prices and model estimates that can be economically exploited with simple buy-and-hold portfolio strategies. Overall, these results suggest our model is a promising pricing tool.

The dynamic model we derive in this article links two different strands of literature, namely, dynamic programming models of the firm and firm valuation models. The former have been used extensively in corporate finance to explain firm behavior.¹ Our model differs from this literature in three fundamental ways, making it particularly useful for asset pricing purposes. First, a problem with that literature is the usual assumption that shareholders are risk-neutral. This assumption results in dynamic programming models of the firm that do not incorporate the risk of the firm and, therefore, would produce biased stock prices if they were used for firm valuation. Alternatively, we invoke the separation principle, which states that managers maximize shareholders' wealth by undertaking the investments that maximize firm value, independently of equity-holders' personal preferences. Thus, our approach does not require any assumption about shareholders' utility functions, as long as we discount future cash flows with an appropriately risk-adjusted discount rate.² Dixit and Pindyck (1994) suggest the possibility of using dynamic programming techniques for firm valuation. We believe our paper is one of the first attempts in this direction.³ Second, virtually all dynamic programming models of the firm in corporate

¹Strebulaev and Whited (2012) provide a comprehensive review of this growing literature.

²See, for example, Copeland, Weston, and Shastri (2005) for a more complete discussion of the separation principle.

³As it is common with other valuation models (e.g., the Black-Scholes formula), we do not introduce adjustment or transaction costs to our model.

finance do not allow firms to grow in the long-run. We overcome this important drawback by introducing secular growth, which could be interpreted as the possibility of the firm to take advantage of new, profitable investments in the future. As documented by Lazzati and Menichini (2014a), long-run growth can account for more than 30% of the value of the firm, and it is of particular importance for certain industries, such as manufacturers of chemical products and industrial machinery, and providers of communication services (Jorgenson and Stiroh (2000)). Third, our model is simpler than most existing dynamic programming models of the firm and we are able to solve it in closed-form, which substantially simplifies its practical implementation.⁴

Our paper is also related to the literature on firm valuation models. Generally accepted valuation models include the dividend discount model (DDM), the discounted cash flow model (DCF), and the residual income model (RIM).⁵ There are two issues that often affect the practical implementation of the aforementioned models. First, those methods typically involve forecasting future values for a finite number of years. The implementation difficulty stems from the fact that the user needs to project sales, costs, and other items that form the cash flows required by the model, and standard valuation models do not offer a clear guide on how to do this task. Second, they also require predicting a terminal or continuation value, which represents the value of the firm at the end of the forecast horizon. This terminal value often represents a large part of the current value of the firm and is usually based on ad hoc calculations. Our model can be regarded as a dynamic version of the DDM. It solves the problems described above by yielding the stock price (i.e., and the future cash flows) as an analytic function of the economic fundamentals of the firm (e.g., the volatility and mean-reversion of profits, the depreciation rate,

⁴A further contribution regarding the literature on dynamic programming models is the explicit description of the role of the parameter representing the drift in the profit process as a regulator of the size of the firm. We explain our contribution regarding this parameter in detail in Section 2.

⁵The DDM is attributed to Williams (1938), while the DCF is described in detail by Copeland, Weston, and Shastri (2005), and Koller, Goedhart, and Wessels (2010), among others. The RIM was originally introduced by Preinreich (1938) and Edwards and Bell (1961), and more recently extended by Ohlson (1991, 1995). While theoretically equivalent, the three valuation models differ with respect to the information used in their practical implementation. The DDM uses the future stream of expected dividend payments to shareholders. The DCF is based on some measure of future cash flows, such as free cash flows. Finally, the RIM uses accounting data (e.g., current and future book value of equity and earnings).

and the elasticity of capital). This analytic solution leads, in turn, to a new approach to firm valuation that involves only two steps. First, model parameters must be calibrated or estimated using historical financial statements, and we offer a clear guide on how to obtain them. Second, using only contemporaneous and observable information, such as the current values of book equity and gross profits, our model estimates the stock price by solving systematically for the infinite sequence of expected future dividends. In other words, the user does not need to estimate either future cash flows or an arbitrary terminal value.⁶

Several papers have studied the empirical performance of the standard pricing models. For instance, Kaplan and Ruback (1995) study the ability of the DCFM to explain the observed market values of 51 highly leveraged transactions. They find that the model produces estimates that are, on average, within 10% of the market prices. Bernard (1995) compares the ability of the DDM and RIM to explain the observed variation in stock prices. He finds that the RIM explains 68% of the variability in market values and outperforms the DDM, which can only explain 29% of such variation. Frankel and Lee (1998) test the RIM empirically and find that the model estimates are highly correlated with current stock prices. Copeland, Weston, and Shastri (2005) test the validity of the DCFM using a sample of 65 firms and find that the model produces value estimates that are quite close to market values. As in these papers, we find that our dynamic valuation model yields equity value estimates that are, on average, very close to market values. Specifically, the median (mean) distance between estimated values and market prices is only 1% (12%). In addition, we regress the market value of equity on the value estimated by the model and find that the latter can explain a large fraction of the variability of the former (around 75%). The main advantage of our approach over that literature is, as we described above, the objectivity and simplicity of its implementation.

While we show that our estimates are, on average, very close to market values —providing some support to the hypothesis of efficient markets in the long-run— we also find temporary deviations between stock prices and model estimates. We then implement simple buy-and-hold portfolio strategies that take advantage of those deviations. The former consist in ranking firms

⁶A further feature of our valuation approach is that it can be easily converted into an n-stage dividend discount model by properly adjusting the future growth rates. In this way, the model can account for different stages of growth in the life of the firm.

based on their ratios of market prices to model estimates, then forming quintile portfolios based on those ratios, and finally buying the firms in the lowest quintile portfolio and selling the firms in the highest quintile portfolio. Our results show that those strategies earn, on average, around 22%, 37%, and 49% returns after one, two, and three years of portfolio formation, respectively. As benchmark, we calculate the returns of portfolios constructed according to the market-to-book ratio and find that they yield around 18%, 28%, and 29% returns in the same periods. That is, our portfolio strategy consistently outperforms that based on the market-to-book ratio.

Finally, it is also relevant to highlight that, while we use publicly available stock prices to assess our model estimates, there are many situations in which market prices are not available. These include, for instance, the valuation of private companies such as Koch Industries and Cargill, the pricing of IPOs such as Facebook in 2012 and Alibaba Group Holding in 2014, and even the assessment of firms' new investment projects. Our method could be used in all these cases, as long as data on financial statements are available.⁷

The paper is organized as follows. In Section 2, we derive a dynamic version of the DDM in closed-form. In Section 3, we describe the data and the calibration of model parameters. The results from the empirical tests of our model are in Section 4. Section 5 concludes. Appendix 1 contains a sensitivity analysis of the valuation results, while Appendix 2 contains the proofs.

2 A Dynamic Dividend Discount Model

In this section, we derive a dynamic version of the standard DDM in closed-form. We solve the problem of the firm (i.e., share price maximization) using discrete-time, infinite-horizon, stochastic dynamic programming within the context of the Adjusted Present Value (APV) method introduced by Myers (1974).

The life horizon of the firm is infinite, which implies that shareholders believe it will run forever. The CEO makes investment and financing decisions at the end of every time period

⁷Complementing the results in this article, Lazzati and Menichini (2014b) show that this model also explains numerous important regularities documented by the empirical literature in corporate finance. For instance, it rationalizes the negative association between profitability and leverage, the existence and characteristics of all-equity firms, and the inverse relation between dividends and investment-cash flow sensitivities.

(e.g., month, quarter, or year) such that the market value of equity is maximized.⁸ (In this paper, we write a tilde on X (i.e., \tilde{X}) to indicate that the variable is growing over time.) Variable \tilde{K}_t represents the book value of assets in period t . The assets of the firm \tilde{K}_t will vary (i.e., increase or decrease) over time, reflecting the investment decisions. In each period, installed capital depreciates at constant rate $\delta > 0$. The debt of the firm in period t , \tilde{D}_t , matures in one period and is rolled over at the end of every period.⁹ We assume the coupon rate c_B equals the market cost of debt r_B , which implies that book value of debt \tilde{D}_t equals market value of debt \tilde{B}_t .¹⁰ The amount of outstanding debt \tilde{B}_t will increase or decrease over time according to financing decisions. Similar to DeAngelo, DeAngelo, and Whited (2011), we assume debt remains risk-free over the firm's life. This assumption aims to capture the phenomenon of debt conservatism documented by Graham (2000). Lazzati and Menichini (2014b) show that this model generates leverage predictions that are consistent with several key results reported by the capital structure literature. In addition, this feature allows us to obtain an analytic solution for the model.

We introduce randomness into the model through the profit shock z_t . It is common in the corporate finance literature to assume that random shocks follow an AR(1) process in logs

$$\ln(z_t) = \ln(c) + \rho \ln(z_{t-1}) + \varepsilon_t \quad (1)$$

where $\rho \in (0, 1)$ is the autoregressive parameter that defines the persistence of profit shocks. In other words, a high ρ makes periods of high profit innovations (e.g., economic expansions) and low profit shocks (e.g., recessions) last more on average, and vice versa. The innovation term ε_t is assumed to be an *iid* normal random variable with mean 0 and variance σ^2 . Constant $c > 0$ is a drift in logs that scales the moments of the distribution of z_t . This parameter has a direct impact on expected profits and, thus, regulates the size of the firm.¹¹ It is common in the corporate finance literature to normalize c to 1, which allows for the study of representative firms but undermines any valuation attempt as the estimated prices result strongly biased. Given that

⁸We use the terms market value of equity, share price, and stock price interchangeably in this paper.

⁹Alternatively, \tilde{D}_t could be interpreted as a perpetuity that the firm increases and decreases as needed at the end of every period.

¹⁰It is straightforward to generalize this component and assume a coupon rate c_B different from the market cost of debt r_B . Without any loss of generality and to simplify notation, we assume they are equal.

¹¹Another way to regulate firm size is to use a constant (e.g., A) as a factor in equation (2).

our objective is to evaluate the pricing performance of this model with actual firms, we do not do such normalization.

Gross profits in period t are defined by the following function

$$\tilde{Y}_t = (1 + g)^{t(1-\alpha)} z_t \tilde{K}_t^\alpha \quad (2)$$

where z_t is the profit shock in period t and parameter $\alpha \in (0, 1)$ represents the elasticity of capital input. The level of technology in period t takes the form of $(1 + g)^{t(1-\alpha)}$, which implies the firm grows at constant rate $g \geq 0$ in each period. With this factor, the firm becomes a scaled up replica of itself over time, and we use this feature in a normalization of growing variables that is required to solve the problem of the firm. Equation (2) says that gross profits also depend on a Cobb-Douglas production function with decreasing returns to scale in capital input.¹²

Every period, the firm pays operating costs $f\tilde{K}_t$ (with $f > 0$) and corporate earnings are taxed at rate $\tau \in (0, 1)$. Therefore, the firm's net profits in period t are

$$\tilde{N}_t = \left(\tilde{Y}_t - f\tilde{K}_t - \delta\tilde{K}_t - r_B\tilde{B}_t \right) (1 - \tau). \quad (3)$$

Finally, the restriction $(f + \delta)(1 - \tau) \leq 1$ guarantees the market value of equity is weakly positive.¹³ With all the previous information, we can state the cash flow that the firm pays to equity-holders in period t as

$$\tilde{L}_t = \tilde{N}_t - \left[\left(\tilde{K}_{t+1} - \tilde{K}_t \right) - \left(\tilde{B}_{t+1} - \tilde{B}_t \right) \right]. \quad (4)$$

Equation (4) implies that the dividend paid to shareholders in period t equals net profits minus the change in equity. We let rate r_S represent the market cost of equity and rate r_A denote the market cost of capital. For existence of the market value of equity, we impose the usual restriction that the secular growth rate must be lower than the market cost of capital (i.e., $g < r_A$). Given

¹²Equation (2) can take on only (weakly) positive values. However, the model can be easily extended to allow for negative values of gross profits by subtracting a random variable as a proportion of assets (e.g., $\eta a \tilde{K}_t$) in equation (2), where η is a Bernoulli random variable and a is a positive constant. While the model still has a closed-form solution, we do not introduce that feature because we do not observe negative profit shocks among the firms in our sample.

¹³From a practical perspective, if we use the present model letting $(f + \delta)(1 - \tau) > 1$, the probability of a negative share price is almost zero for standard values of the parameters.

the current state of the firm at $t = 0$, $(\tilde{K}_0, \tilde{B}_0, z_0)$, the problem of the CEO is to choose an infinite sequence of functions $\{\tilde{K}_{t+1}, \tilde{B}_{t+1}\}_{t=0}^{\infty}$, such that the market value of equity is maximized. We let E_0 indicate the expectation operator given information at $t = 0$ (i.e., $\tilde{K}_0, \tilde{B}_0, z_0$). The stock price can thereby be expressed as

$$\tilde{S}_0(\tilde{K}_0, \tilde{B}_0, z_0) = \max_{\{\tilde{K}_{t+1}, \tilde{B}_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \frac{1}{\prod_{j=0}^t (1 + r_{S_j})} \tilde{L}_t \quad (5)$$

subject to the restriction of risk-free debt. Formally, we say debt is risk-free if, in every period, the after-shock book value of equity is weakly positive. In other words, net profits plus the sale of assets, $\tilde{N}_t + \tilde{K}_t$, must be sufficient to cover debt, \tilde{B}_t . This condition is equivalent to a weakly positive net-worth covenant. This type of covenant is often used with short-term debt contracts (see, e.g., Leland (1994)), and fits nicely with the one-period debt in our model.

We solve the problem in equation (5) and find the following closed-form for the stock price.

Proposition 1 *The market value of equity is*

$$\tilde{S}_t(\tilde{K}_t, \tilde{B}_t, z_t) = \left[(1+g)^{t(1-\alpha)} z_t \tilde{K}_t^\alpha - f \tilde{K}_t - \delta \tilde{K}_t - r_B \tilde{B}_t \right] (1-\tau) + \tilde{K}_t - \tilde{B}_t + \tilde{G}_t(z_t) \quad (6)$$

where the going concern value is $\tilde{G}_t(z_t) = \tilde{M}_t(z_t) P^*$. Variable $\tilde{M}_t(z_t)$ is given by

$$\tilde{M}_t(z_t) = (1+g)^t e^{-\frac{1}{2}\sigma^2 \frac{\alpha}{(1-\alpha)^2}} \sum_{n=1}^{\infty} \left(\frac{1+g}{1+r_A} \right)^n E \left[z_{t+n}^{1/(1-\alpha)} | z_t \right] \quad (7)$$

with the general term

$$E \left[z_{t+n}^{1/(1-\alpha)} | z_t \right] = \left(c^{\frac{1-\rho^n}{1-\rho}} z_t^{\rho^n} e^{\frac{1}{2}\sigma^2 \frac{(1-\rho^{2n})}{(1-\rho^2)} \frac{1}{(1-\alpha)}} \right)^{\frac{1}{1-\alpha}}, \quad n = 1, 2, \dots \quad (8)$$

and variable P^* takes the form

$$P^* = (W^{*\alpha} - fW^* - \delta W^*) (1-\tau) - r_A W^* + \left(\frac{1+r_A}{1+r_B} \right) r_B \tau \ell^* W^* \quad (9)$$

with

$$W^* = \left(\frac{\alpha}{\frac{r_A}{1-\tau} + f + \delta} \right)^{\frac{1}{1-\alpha}} \quad \text{and} \quad \ell^* = \frac{1 - (f + \delta)(1-\tau)}{1 + r_B(1-\tau)}. \quad (10)$$

The proof of Proposition 1 is in Appendix 2. Lazzati and Menichini (2014a) provide a detailed description of each of their components.

The market value of equity is shown in equation (6) and represents an analytic solution of the Gordon Growth Model in the dynamic and stochastic setting. The first three terms in equation (6) represent the after-shock book value of equity, while the last term, $\widetilde{G}_t(z_t)$, is the going-concern value. The latter depends on variable $\widetilde{M}_t(z_t)$, which captures the effect of the infinite sequence of expected profit shocks, and on variable P^* , which denotes the dollar return on capital minus the dollar cost of capital at the optimum (plus the interest tax shields).¹⁴ The going-concern value shows that our model does not require the forecast of future values or the computation of a terminal value. Using only information about the current state (i.e., current book value of equity and gross profits), our model solves systematically for the full sequence of expected future dividends. Furthermore, function $\widetilde{M}_t(z_t)$ suggests that our model can become an n-stage dynamic DDM if we substitute the growth rate g on the numerator of the discount factor appropriately.¹⁵

In the next section, we describe our sample and the calibration of model parameters. We subsequently use this information to confirm the empirical validity of our valuation method.

3 Data and Calibration of Model Parameters

Our original sample contains the 100 firms included in the S&P 100 Index in March 2014. We construct this sample using two data sources. Historical accounting data are obtained from the Compustat annual files, while the corresponding stock price data are obtained from the CRSP

¹⁴We obtain $\widetilde{M}_t(z_t)$ in the following way. Let $A_0 = 0$ and, for $n = 1, 2, \dots$,

$$A_n = A_{n-1} + \left(\frac{1+g}{1+r_A} \right)^n E \left[z_{t+n}^{1/(1-\alpha)} | z_t \right].$$

Then, we iterate the previous recursion until convergence (i.e., until $A_n = A_{n-1} = A$). Finally, we compute $\widetilde{M}_t(z_t)$ as

$$\widetilde{M}_t(z_t) = (1+g)^t e^{-\frac{1}{2}\sigma^2 \frac{\alpha}{(1-\alpha)^2}} A.$$

¹⁵Equation (6) only includes the interest tax shields as financing side effects. The original formulation of APV in Myers (1974) also allows for other components of the financing side effects, such as the issue costs of new securities and the costs of financial distress.

monthly files. The sample covers the period 1990-2013. For most of those firms (e.g., Coca-Cola and Boeing) we obtain data from Compustat and CRSP from 1990 through 2013. For some newer firms we obtain data starting in a later year (e.g., 2002-2013 for Google and 2007-2013 for Facebook). We use the historical data for each firm to estimate its own parameters (e.g., we employ the data on Caterpillar to obtain the parameters to be used in the valuation of Caterpillar and so on for every other firm.) In all our empirical analyses, we ensure that accounting data are known at the time the stock price is set in the exchange. Thus, we use the share price observed five months later than the fiscal-year-end of the firm. For instance, we match the accounting data of a December year-end firm with its closing stock price at the end of May of the following year.¹⁶

We use the data from the previous sample to calibrate parameters $c, \rho, \sigma, \alpha, f, \delta, \tau, r_B, r_A$, and g for each firm in the S&P 100 Index. In order to obtain parameter f , we average the ratio Selling, General, and Administrative Expense (XSGA)/Assets - Total (AT) for each firm. We follow the same procedure to get δ as the ratio of Depreciation and Amortization (DP) over Assets - Total (AT), and τ as the fraction Income Taxes - Total (TXT)/Pretax Income (PI). Similar to Moyen (2004), we obtain parameters c, ρ, σ , and α for each company using the firm's autoregressive profit shock process in equation (1) and the gross profits equation (2). The data we use with these equations are Gross Profit (GP) and Assets - Total (AT). We use a risk-free interest rate ($r_f = r_B$) of 0.03 for all firms, which is close to the average 1-year T-Bill yield for the sample period. We follow the procedure described by Kaplan and Ruback (1995) to obtain the market cost of capital. That is, we derive r_A using CAPM after unlevering the equity beta and assuming an expected market risk premium of 0.06. Finally, we assume the long-run growth rate g is 0.03 for all firms.¹⁷

After calibrating model parameters, we check the consistency of their values with the assumptions made by the model in Section 2. We find that some firms have an elasticity of capital (α) above one, while the model constrains that parameter to be between zero and one (i.e., decreasing

¹⁶Regarding the buy-and-hold strategy, all firms are present every time we purchase or sell them (i.e., no firm leaves the sample while the strategy is in place). Therefore, we believe the sample does not suffer from a survivorship bias problem.

¹⁷Computing parameter g for each firm would probably yield better results than those reported in the present study. In addition, using the model as an n-stage dynamic DDM could improve results even more.

returns to scale in capital input in the production function in equation (2)). We exclude these firms, which add up to 40. We do not find problems with the other parameters. In addition, we exclude firms with missing data and financial firms (i.e., SIC between 6000 and 6999). After these filters, the final sample includes 1,035 firm-year observations, which implies an average of 45 firms per year.

4 Empirical Results

In this section, we study two different aspects of the dynamic model we offer. First, we analyze the consistency between the model estimates and market prices (i.e., how close are the predicted values by our model to the actual market prices on average). Complementing that analysis, we examine how much of the observed variation in contemporaneous share prices is explained by the model estimates. Second, we investigate the possibility to use our model to predict future stock returns and, thus, exploit the differences between those two values.

4.1 Explanation of Contemporaneous Stock Prices

We start studying the consistency between model estimates and market prices. To this end, for each firm-year observation in the sample, we construct the market-to-value ratio (P/V), which is the market value of equity (P) divided by the equity value estimated by the dynamic DDM (V). Table I reports the summary statistics for this ratio. Panel A shows that the median of P/V is 1.01, which is quite close to the desired value of 1. This result implies that the median observation of the market value of equity is 1% larger than the median dynamic DDM estimate. The mean of P/V has a value of 1.12. We believe the difference between the mean and the median is reasonable because the market-to-value ratio is bounded below at zero but unbounded above, which creates a distribution of P/V that is skewed to the right. Panel B in Table I presents different measures of central tendency. For instance, it shows that 24.09% of the estimated values are within 15% of the market value of equity. Finally, the second column in Table I presents the same statistics using the log of P/V , which could be interpreted as the percentage difference between the market value of equity and the value estimated by the model. These results turn out to be close to those

of Kaplan and Ruback (1995), who do a similar analysis to assess the empirical performance of the DCFM in the context of highly leveraged transactions.¹⁸

[Insert Table I here]

Figure 1 shows the evolution of the mean value of the market-to-value ratio (line with crosses) over the period 1990-2013. In accordance with the results in Table I, the ratio is close to the value of 1 the whole period. As benchmark, we add the mean market-to-book ratio (line with solid squares), which is the market value of equity (P) divided by the book value of equity (B). Both lines attain their maximum values at the end of the 1990s, around the culmination of the stock market surge. It is also clear the impact of the market crash in 2008 on mean P/V , shifting it toward 1.

[Insert Figure 1 here]

We close this subsection analyzing the association between our model estimates and contemporaneous stock prices. That is, we ascertain the proportion of the variation in current prices that is explained by the predictions of the dynamic DDM. Accordingly, we estimate the following basic model

$$P_{i,t} = \alpha + \beta V_{i,t} + \epsilon_{i,t} \tag{11}$$

where P denotes the market value of equity and V represents the value estimated by the model. In this specification, i indexes firms, t indexes time periods, and $\epsilon_{i,t}$ is an *iid* random term. In theory, an intercept of zero and a slope of one imply that our model produces unbiased estimates of market values.

The first column in Table II shows the results from this regression. We cannot reject the hypothesis that the intercept is zero ($t = 1.16$). In addition, the slope is very close to one

¹⁸Appendix 1 contains a sensitivity analysis of these valuation results with respect to the growth rate and the equity risk premium.

($\beta = 1.02$) and statistically significant ($t = 49.90$). With an r-squared of 75.8%, the dynamic DDM explains a large proportion of the variation in current stock prices.

[Insert Table II here]

We finally explore the role of the different parts of the dynamic DDM in explaining the variability of contemporaneous stock prices. Proposition 1 shows that the stock price estimated by the model can be separated in the after-shock book value of equity and the going-concern value. Accordingly, we estimate the following model

$$P_{i,t} = \alpha + \beta_1 B_{i,t} + \beta_2 G_{i,t} + \epsilon_{i,t} \quad (12)$$

where B denotes the after-shock book value of equity in our model (which we equate to the book value of equity in our sample), and G represents the going-concern value in our model, which is given by the last term in equation (6). The results from this regression are in the second column of Table II. The intercept is not significantly different from zero ($t = 0.78$) and both regressors have statistically significant coefficients that are relatively close to one. These results suggest that those model components are important determinants of the stock price. In addition, the r-squared from this regression increases to 77.8%.¹⁹

Overall, our valuation results suggest that the dynamic DDM produces equity value estimates that are consistent with market prices and explain a large part of the variation in current stock prices. We next explore the possibility to use the model to exploit the differences between market and estimated values.

4.2 Forecast of Future Stock Returns

In the previous subsection, we showed that the dynamic DDM produces value estimates that are, on average, very close to contemporaneous share prices. Nevertheless, the fact that our model

¹⁹As benchmark, the third column in Table II displays the results from regressing the market value of equity on the book value of equity. As usual, the intercept is significantly different from zero ($t = 11.68$). In addition, while statistically significant ($t = 30.08$), the slope ($\beta = 2.11$) is far from one. Consistently with previous studies, the r-squared is 53.2%.

does not explain 100% of the variation in current stock prices shows evidence of temporary or short-run deviations between market prices (P) and estimated values (V) for individual stocks. In this subsection, we show that these differences can be economically exploited.

To achieve our goal, we first construct portfolios based on the ranking of (demeaned) P/V of the firms in the sample.²⁰ We then form quintile portfolios where lower quintiles include firms with low P/V and higher quintiles include firms with high P/V . Firms in the lower quintile portfolios are, in principle, *undervalued* and are, therefore, expected to experience higher stock returns in the near future. The opposite is true for firms in the higher quintile portfolios. The last step consists in implementing a buy-and-hold strategy and involves taking a long position in the bottom quintile portfolio and a short position in the top quintile portfolio.²¹

Panel A in Table III displays the results of the just described buy-and-hold strategy. The column labeled Q1-Q5 shows that the latter earns 22.40%, 36.95%, and 49.09% on average over the 12, 24, and 36 months following the portfolio formation, respectively. The following column shows that those returns are statistically significant. In the eighth column we exhibit the same returns, but adjusted by the risk implied by the strategy (i.e., the ratio return/standard deviation of returns). The last column reports the percentage of periods in which the strategy earned positive returns. Specifically, it shows that in 95% of the periods (i.e., 19 out of 20 years), the strategy produced positive returns after 36 months of portfolio formation. Overall, these results provide evidence that differences between market prices (P) and our estimated values (V) can indeed be profitable exploited.

To appreciate the magnitude of our previous results, we compare them with a buy-and-hold strategy based on the market-to-book ratio (P/B), which is one of the most well-known accounting-based ratios that exhibit predictive power for stock returns (Fama and French, 1992).²² Panel B of Table III shows that the buy-and-hold strategy using P/V considerably outperforms

²⁰We demean P/V of each firm to eliminate the effect of systematic differences between the market and the estimated value of equity.

²¹To evaluate this strategy, we form quintile portfolios from 1990 through 2009 (i.e., 20 periods) and track the cumulative returns of the strategy over the following 36 months. We restrict the sample to December year-end firms so that the financial information of all the included firms correspond to the same moment. The resulting subsample represents roughly 90% of the original sample.

²²An analogue comparison appears in Frankel and Lee (1998).

the P/B strategy in each of the three investment horizons. For example, over the period of 36 months, the former yields roughly 20% more than the latter (49.09% versus 29.10%) on average. Column 8 shows that this result still holds if we adjust the corresponding returns in terms of risk.²³ Finally, the t -statistics and the percentage of winner periods of the P/B portfolios are, in general, lower than those of the strategy with the market-to-value ratio.

[Insert Table III here]

To highlight our previous results, Figure 2 graphically displays the evolution of the average returns of the P/V and P/B portfolios over the 36 months after portfolio formation. Overlaying the two lines are fitted curves that display the general trends, and corroborate our previous observations. The concavity or flattening of the general trends reflect the fact that the benefits from the information available at the moment of portfolio formation diminish as time passes.

[Insert Figure 2 here]

5 Conclusion

We derive a dynamic version of the dividend discount model (DDM) in closed-form and evaluate its empirical performance. The implementation of our method relies on widely available financial data and implies a new valuation approach that involves two simple steps. First, model parameters (i.e., the proxies for the economic fundamentals of the firm) must be calibrated or estimated. Second, the model uses current data on book value of equity and gross profits to determine the stock price. It does so by systematically projecting the infinite sequence of future expected dividends. Thus, our model does not require to actually forecast any future value (including a terminal value), helping to reduce the degree of discretion on the user side.

²³To evaluate the robustness of this result, we also computed two other common measures of return-to-variability—the Sharpe and Treynor ratios. We obtained analogue results.

The empirical evaluation of the dynamic DDM yields promising results. First, we find that our model forecasts stock prices consistently, that is, model estimates are very close to market prices on average. Second, the model explains a large proportion (around 75%) of the observed variability in current stock prices. Finally, we find that the model can be used to predict future stock returns in the cross-section of firms by exploiting temporary differences between market prices and model estimates. For instance, constructing portfolios based on the ratio of the market value of equity to the equity value estimated by the model, we find that simple buy-and-hold strategies earn considerable positive returns over the three following years (e.g., an average of around 22%, 37%, and 49% returns after 1, 2, and 3 years, respectively, of portfolio formation).

References

- [1] Bernard, Victor L., 1995, The Feltham–Ohlson framework: implications for empiricists, *Contemporary Accounting Research* 11, 733–747.
- [2] Copeland, Thomas E., John F. Weston, and Kuldeep Shastri, 2005, *Financial theory and corporate policy*, Pearson Addison Wesley.
- [3] DeAngelo, Harry, Linda DeAngelo, and Toni M. Whited, 2011, Capital structure dynamics and transitory debt, *Journal of Financial Economics* 99, 235–261.
- [4] Dixit, Avinash K., and Robert S. Pindyck, 1994, *Investment under uncertainty*, Princeton University Press.
- [5] Edwards, Edgar O., and Philip W. Bell, 1961, *The theory and measurement of business income*, University of California Press, Berkeley, CA.
- [6] Fama, Eugene F., and Kenneth R. French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3–56.
- [7] Frankel, Richard, and Charles M. C. Lee, 1998, Accounting valuation, market expectation, and cross-sectional stock returns, *Journal of Accounting and Economics* 25, 283–319.
- [8] Graham, John R., 2000, How big are the tax benefits of debt?, *Journal of Finance* 55, 1901–1941.
- [9] Jorgenson, Dale W. and Kevin J. Stiroh, 2000, U.S. economic growth at the industry level, *American Economic Review* 90, 161–167.
- [10] Kaplan, Steven N., and Richard S. Ruback, 1995, The valuation of cash flow forecasts: an empirical analysis, *Journal of Finance* 50, 1059–1093.
- [11] Koller, Tim, Marc Goedhart, and David Wessels, 2010, *Valuation. Measuring and managing the value of companies*, John Wiley & Sons.
- [12] Lazzati, Natalia, and Amilcar A. Menichini, 2014a, A dynamic approach to the dividend discount model, *Review of Pacific Basin Financial Markets and Policies* 18, 1–31.

- [13] Lazzati, Natalia, and Amilcar A. Menichini, 2014b, A dynamic model of the firm: structural explanations of key empirical findings, *The Financial Review*, Forthcoming.
- [14] Leland, Hayne E., 1994, Corporate debt value, bond covenants, and optimal capital structure, *Journal of Finance* 49, 1213–1252.
- [15] Moyen, Nathalie, 2004, Investment-cash flow sensitivities: Constrained versus unconstrained firms, *Journal of Finance* 59, 2061–2092.
- [16] Myers, Stewart C., 1974, Interactions of corporate financing and investment decisions-implications for capital budgeting, *Journal of Finance* 29, 1–25.
- [17] Ohlson, James A., 1991, The theory of value and earnings, and an introduction to the Ball-Brown analysis, *Contemporary Accounting Research* 7, 1–19.
- [18] Ohlson, James A., 1995, Earnings, book values, and dividends in security valuation, *Contemporary Accounting Research* 11, 661–687.
- [19] Preinreich, Gabriel A. D., 1938, Annual survey of economic theory: the theory of depreciation, *Econometrica* 6, 219–241.
- [20] Strebulaev, Ilya A., and Toni M. Whited, 2012, Dynamic models and structural estimation in corporate finance, *Foundations and Trends in Finance* 6, 1–163.
- [21] Williams, John B., 1938, *The theory of investment value*, Harvard University Press.

6 Appendix 1: Sensitivity Analysis

In this appendix, we discuss the valuation results in the context of a sensitivity analysis with respect to the equity risk premium and the growth rate. These are the only two parameters for which we choose specific values for all firms.

Panel A in Table IV shows the results for the base case with a growth rate of 0.03 and an equity risk premium of 0.06. Panel B suggests that reducing the equity risk premium to 0.04 makes the median value of P/V decrease to 0.64, while increasing the former to 0.08 makes the latter go up to 1.36. The last column shows that the median absolute error increases when we change the equity risk premium from the base case in any direction (i.e., up or down). That is, the median absolute error becomes a minimum close to the value of 0.06 for the equity risk premium.

Panel C in Table IV displays the effects of varying the growth rate. A reduction in the growth rate increases the median market-to-value ratio monotonically, and vice versa. As with the equity risk premium, changing the base case growth rate in any direction increases the median absolute error. Overall, these results suggest that our choice of the base case values for these parameters is reasonable.

[Insert Table IV here]

7 Appendix 2: Proofs

The proof of Proposition 1 requires an intermediate result that we present next.

Lemma 2 *Restricting debt to be risk-free, the maximum level of book leverage in each period is given by*

$$\ell^* = \frac{1 - (f + \delta)(1 - \tau)}{1 + r_B(1 - \tau)}. \quad (13)$$

Proof We say debt is risk-free if, in every period, the following inequality is true for all z'

$$(z'K'^{\alpha} - fK' - \delta K' - r_B \ell K')(1 - \tau) + K' - \ell K' \geq 0. \quad (14)$$

That is, risk-free debt implies that next-period, after-shock book value of equity must be weakly positive for all z' .²⁴ In other words, net profits, $(z'K'^{\alpha} - fK' - \delta K' - r_B \ell K')(1 - \tau)$, plus the sale of assets, K' , must be sufficient to cover debt, $\ell K'$.

Given that the worst-case scenario is $z' = 0$, the maximum book leverage ratio consistent with risk-free debt, ℓ^* , satisfies

$$(-fK' - \delta K' - r_B \ell^* K')(1 - \tau) + K' - \ell^* K' = 0. \quad (15)$$

Working on the previous expression, we can derive the maximum level of book leverage as

$$\ell^* = \frac{1 - (f + \delta)(1 - \tau)}{1 + r_B(1 - \tau)} \quad (16)$$

which completes the proof.²⁵ ■

Proof of Proposition 1

The maximization in equation (5) requires a normalization of growing variables that keeps the expectation of the payoff function in the future periods bounded. This normalization is equivalent to the one used to find the solution of the canonical Gordon Growth Model. Let vector $\tilde{X}_t = \{\tilde{K}_t, \tilde{B}_t, \tilde{Y}_t, \tilde{N}_t, \tilde{L}_t, \tilde{S}_t\}$ contain the growing variables of the model. We then transform vector \tilde{X}_t

²⁴The same result is true if we define risk-free debt using the market value of equity as opposed to the book value of equity. That is, in both cases we arrive at equation (13) as the maximum level of book leverage consistent with risk-free debt. In order to simplify notation, we use the book value of equity.

²⁵The restriction $(f + \delta)(1 - \tau) \leq 1$ described in Section 2 also guarantees that $\ell^* \geq 0$.

in the following way: $X_t = \tilde{X}_t/(1+g)^t$. Using the normalized variables and modifying the payoff function accordingly, the market value of equity can be expressed as

$$S_0(K_0, B_0, z_0) = \max_{\{K_{t+1}, B_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \frac{(1+g)^t}{\prod_{j=0}^t (1+r_{S_j})} L_t \quad (17)$$

subject to keeping debt risk-free. Because we use the Adjusted Present Value method of firm valuation, we solve the problem of the firm in equation (17) in three steps. First, we determine the value of the unlevered firm, $S_{u_0}(K_0, z_0)$. Second, we solve for optimal debt and compute the present value of the financing side effects. Finally, we obtain the value of the levered firm in equation (17).

The market value of equity for the unlevered firm can be expressed as

$$S_{u_0}(K_0, z_0) = \max_{\{K_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \left(\frac{1+g}{1+r_A} \right)^t L_{u_t} \quad (18)$$

where $L_{u_t} = N_{u_t} - (K_{t+1} - K_t)$ and $N_{u_t} = (Y_t - fK_t - \delta K_t)(1 - \tau)$. We let normalized variables with primes indicate values in the next period and normalized variables with no primes denote current values. Then, the Bellman equation for the problem of the firm in equation (18) is given by

$$S_u(K, z) = \max_{K'} \left\{ (zK^\alpha - fK - \delta K)(1 - \tau) - (1+g)K' + K + \frac{(1+g)}{(1+r_A)} E[S_u(K', z') | z] \right\}. \quad (19)$$

We use the guess and verify method as the proof strategy. Thus, we start by guessing that the solution is given by

$$S_u(K, z) = (zK^\alpha - fK - \delta K)(1 - \tau) + K + M(z) P_u^* \quad (20)$$

where

$$M(z) = e^{-\frac{1}{2}\sigma^2 \frac{\alpha}{(1-\alpha)^2}} \sum_{n=1}^{\infty} \left\{ \left(\frac{1+g}{1+r_A} \right)^n \left(c \frac{1-\rho^n}{1-\rho} z^{\rho^n} e^{\frac{1}{2}\sigma^2 \frac{(1-\rho^{2n})}{(1-\rho^2)} \frac{1}{(1-\alpha)}} \right)^{\frac{1}{1-\alpha}} \right\}, \quad (21)$$

$$P_u^* = (W^{*\alpha} - fW^* - \delta W^*)(1 - \tau) - r_A W^*, \quad (22)$$

$$W^* = \left(\frac{\alpha}{\frac{r_A}{1-\tau} + f + \delta} \right)^{\frac{1}{1-\alpha}}. \quad (23)$$

We obtained this initial guess as the solution of equation (19) by the backward induction method.

We now verify our guess. To this end, let us write

$$S_u(K, z) = \max_{K'} \{F(K', K, z)\} \quad (24)$$

with F defined as the objective function in equation (19).

The FOC for this problem is

$$\partial F(K', K, z) / \partial K' = -(1+g) + \frac{(1+g)}{(1+r_A)} [(E[z'|z] \alpha K'^{\alpha-1} - f - \delta)(1-\tau) + 1] = 0 \quad (25)$$

and optimal capital turns out to be

$$K^* = E[z'|z]^{\frac{1}{1-\alpha}} W^* \quad (26)$$

where W^* is as in equation (23).

Finally, the market value of equity for the unlevered firm becomes

$$\begin{aligned} S_u(K, z) &= (zK^\alpha - fK - \delta K)(1-\tau) - (1+g)K^* + K + \\ &\quad \frac{(1+g)}{(1+r_A)} [(E[z'|z] K^{*\alpha} - fK^* - \delta K^*)(1-\tau) + K^* + \\ &\quad E[M(z')|z] P_u^*] \\ &= (zK^\alpha - fK - \delta K)(1-\tau) + K - (1+g)E[z'|z]^{\frac{1}{1-\alpha}} W^* + \\ &\quad \frac{(1+g)}{(1+r_A)} \left\{ E[z'|z]^{\frac{1}{1-\alpha}} [(W^{*\alpha} - fW^* - \delta W^*)(1-\tau) + W^*] + \right. \\ &\quad \left. E[M(z')|z] P_u^* \right\} \\ &= (zK^\alpha - fK - \delta K)(1-\tau) + K + \\ &\quad \frac{(1+g)}{(1+r_A)} \left(e^{-\frac{1}{2}\sigma^2 \frac{\alpha}{(1-\alpha)^2}} E[z'^{\frac{1}{1-\alpha}}|z] + E[M(z')|z] \right) P_u^* \\ &= (zK^\alpha - fK - \delta K)(1-\tau) + K + M(z) P_u^* \end{aligned} \quad (27)$$

which is equivalent to our initial guess in equation (20).

Next, we obtain optimal debt. In each period, the firm solves the following problem

$$B^* = \max_{B'} \left\{ B' - \frac{1}{(1+r_B)} B' [1 + r_B(1-\tau)] \right\} \quad (28)$$

subject to the restriction of risk-free debt. Because $\tau > 0$, the firm increases debt as much as possible (as long as it remains risk-free) in order to maximize the tax benefits of debt. Then, optimal debt is $B^* = \ell^* K^*$ where

$$\ell^* = \frac{1 - (f + \delta)(1-\tau)}{1 + r_B(1-\tau)}, \quad (29)$$

as shown in Lemma 2. The present value of the financing side effects turns out to be

$$\begin{aligned} Q(z) &= \left(\frac{1+g}{1+r_A}\right) \left\{ \left(\frac{1+r_A}{1+r_B}\right) r_B \tau B^* + E[Q(z')|z] \right\} \\ &= M(z) \left(\frac{1+r_A}{1+r_B}\right) r_B \tau \ell^* W^* \end{aligned} \quad (30)$$

where $M(z)$ is as in equation (21). Under this financial policy, the amount of debt and interest payments will vary with the future asset cash flows (i.e., they depend on future firm performance). Then, because future interest tax shields will have a level of risk in line with that of the firm cash flows, we use the cost of capital, r_A , as the discount rate.

The third step consists in obtaining the market value of equity for the levered firm. If we assume the firm used debt B in the previous period, and now has to pay interest $r_B B(1 - \tau)$, then the stock price for the levered firm is

$$\begin{aligned} S(K, B, z) &= S_u(K, z) + M(z) \left(\frac{1+r_A}{1+r_B}\right) r_B \tau \ell^* W^* - B - r_B B(1 - \tau) \\ &= (zK^\alpha - fK - \delta K - r_B B)(1 - \tau) + K - B + G(z) \end{aligned} \quad (31)$$

where $G(z) = M(z)P^*$ and variable P^* takes the form

$$P^* = (W^{*\alpha} - fW^* - \delta W^*)(1 - \tau) - r_A W^* + \left(\frac{1+r_A}{1+r_B}\right) r_B \tau \ell^* W^*. \quad (32)$$

The last part of the proof consists in transforming normalized variables back into growing variables. For this step, we return to the initial notation with growing variables, where next-period assets are \tilde{K}_{t+1} and current-period assets are \tilde{K}_t . Then, the required transformation is: $\tilde{X}_t = (1+g)^t X_t$, where vector $X_t = \{K_t, B_t, Y_t, N_t, L_t, S_t\}$ contains the normalized variables of the model. Finally, the optimal decisions of the firm with growing variables are given by

$$\tilde{K}_{t+1}^*(z_t) = (1+g)^{t+1} E[z_{t+1}|z_t]^{\frac{1}{1-\alpha}} W^* \quad \text{and} \quad \tilde{B}_{t+1}^*(z_t) = \ell^* \tilde{K}_{t+1}^*(z_t) \quad (33)$$

and the growing market value of equity is

$$\tilde{S}_t(\tilde{K}_t, \tilde{B}_t, z_t) = \left[(1+g)^{t(1-\alpha)} z_t \tilde{K}_t^\alpha - f\tilde{K}_t - \delta\tilde{K}_t - r_B \tilde{B}_t \right] (1 - \tau) + \tilde{K}_t - \tilde{B}_t + \tilde{G}_t(z_t) \quad (34)$$

as shown in Proposition 1. ■

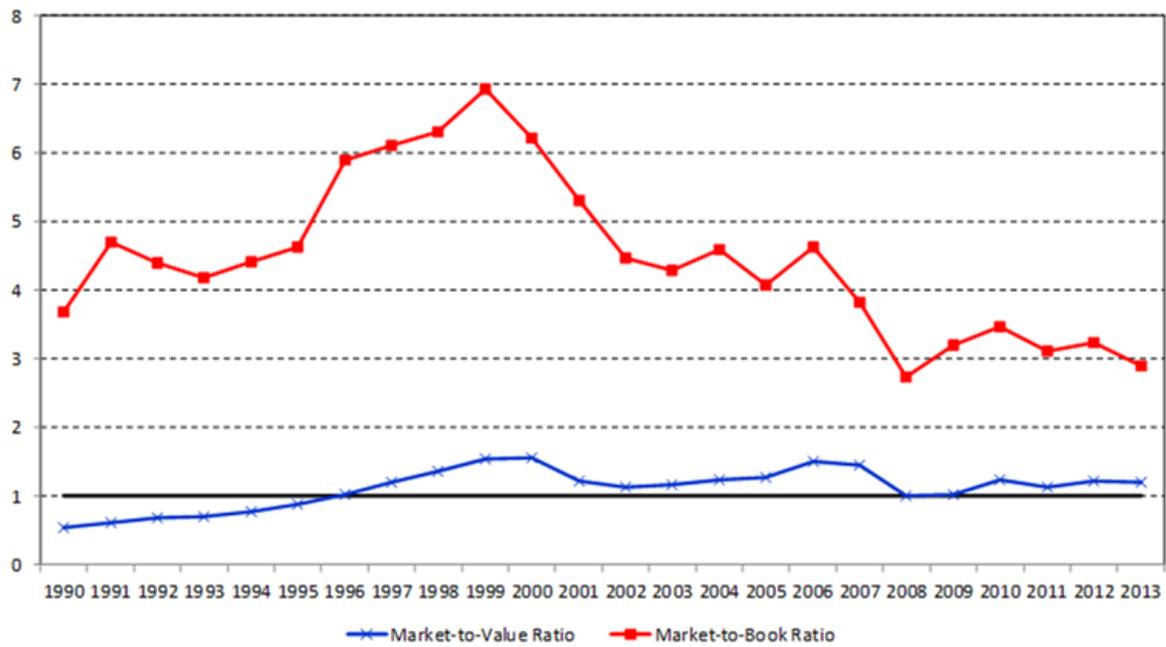


Figure 1. Evolution of the market-to-value ratio. The figure displays the evolution over time of the mean market-to-value ratio (line with crosses) and the mean market-to-book ratio (line with solid squares) for a sample of firms included in the S&P 100 Index as of March 2014. The sample covers the period 1990-2013. The market-to-value ratio is the market value of equity divided by the value estimated by the model. The market-to-book ratio is the market value of equity divided by the book value of equity.

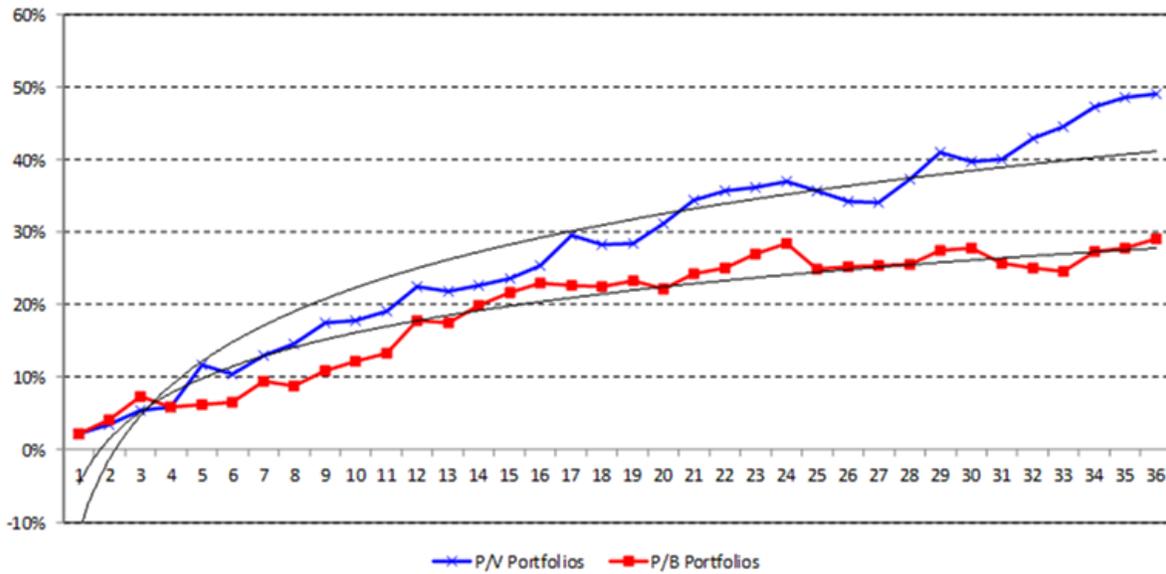


Figure 2. Cumulative buy-and-hold returns. The figure displays the cumulative returns of the buy-and-hold strategies for the P/V and P/B portfolios during the 36 months following portfolio formation. These portfolios are constructed with a sample of firms included in the S&P 100 Index as of March 2014. The sample covers the period 1990-2013. P/V is the market-to-value ratio (i.e., the market value of equity divided by the equity value estimated by the model). P/B is the market-to-book ratio (i.e., the market value of equity divided by the book value of equity). Portfolios are formed by sorting firms into quintiles according to their P/V and P/B ratios at the end of May of each year. The buy-and-hold strategy consists in buying firms in the bottom quintile and selling firms in the top quintile.

Table I
Valuation Results

The table shows the valuation results of the dynamic DDM for a sample of firms included in the S&P 100 Index as of March 2014. The sample covers the period 1990-2013. P/V is the market-to-value ratio (i.e., the market value of equity divided by the equity value estimated by the model). The $\log(P/V)$ results derive from computing the log of the market-to-value ratio. The first line in Panel B shows the percentage of times that the value estimated by the model is within 15% of the market value of equity. The second line in Panel B shows the median value of the absolute difference between the equity value estimated by the model and the market value of equity (in percent). The third line in Panel B shows the median value of the squared difference between the value estimated by the model and the market value of equity (in percent). t -statistics are in parentheses.

	P/V	$\log(P/V)$
<i>Panel A: Summary Statistics</i>		
<i>Mean</i>	1.12 (45.85)	-3.41% (-3.20)
<i>Median</i>	1.01	0.22%
<i>Standard Deviation</i>	0.56	25.37%
<i>Interquartile Range</i>	0.68	30.04%
<i>Panel B: Performance Measures</i>		
<i>Percentage within 15%</i>	24.09%	49.44%
<i>Median Absolute Error</i>	33.15%	15.25%
<i>Median Squared Error</i>	11.46%	2.32%

Table II
Regression of the Market Value of Equity

The table shows the results from different cross-sectional regressions of the market value of equity. In column (1), the regressor is the equity value estimated by the model. In column (2), the regressors are the book value of equity and the going-concern value estimated by the model. In column (3), the regressor is the book value of equity. The sample is composed of firms included in the S&P 100 Index as of March 2014, and covers the period 1990-2013. *t*-statistics are in parentheses.

<i>Variable</i>	<i>(1)</i>	<i>(2)</i>	<i>(3)</i>
<i>Intercept</i>	1574.30 (1.16)	1018.43 (0.78)	19307.16 (11.68)
<i>Equity Value Estimate</i>	1.02 (49.90)		
<i>Book Value of Equity</i>		1.44 (27.11)	2.11 (30.08)
<i>Going-Concern Value Estimate</i>		0.84 (29.69)	
<i>R</i> ²	0.758	0.778	0.532

Table III
Cumulative Buy-and-Hold Returns of Quintile Portfolios

The table presents the cumulative buy-and-hold returns of three different strategies. *P/V* Portfolios is the strategy that constructs portfolios based on the ranking of the market-to-value ratio (*P/V*). *P/B* Portfolios is the strategy that constructs portfolios based on the ranking of the market-to-book ratio (*P/B*). Hybrid Portfolios is the strategy that constructs portfolios based on mixing the previous two in equal parts (i.e., 50% of the *P/V* Portfolios and 50% of the *P/B* Portfolios). *Ret12*, *Ret24*, and *Ret36* are the average 12-month, 24-month, and 36-month buy-and-hold returns of each strategy, respectively. *Q1-Q5* is the average spread of returns between the lowest (*Q1*) and highest (*Q5*) quintile portfolios. *Ret/Risk* is the ratio of the portfolio return to its standard deviation of returns. % *Winners* is the percentage of periods (out of 20) in which the strategy yielded positive returns.

	<i>Q1</i>	<i>Q2</i>	<i>Q3</i>	<i>Q4</i>	<i>Q5</i>	<i>Q1-Q5</i>	<i>t-statistic</i>	<i>Ret/Risk</i>	% <i>Winners</i>
<i>Panel A: P/V Portfolios</i>									
<i>Ret12</i>	20.34%	7.33%	1.72%	0.07%	-2.06%	22.40%	4.48	1.00	90.00%
<i>Ret24</i>	31.60%	16.23%	4.06%	0.25%	-5.35%	36.95%	5.79	1.29	90.00%
<i>Ret36</i>	42.13%	17.77%	10.09%	0.19%	-6.96%	49.09%	5.42	1.21	95.00%
<i>Panel B: P/B Portfolios</i>									
<i>Ret12</i>	14.30%	8.13%	4.97%	4.73%	-3.50%	17.80%	3.54	0.79	75.00%
<i>Ret24</i>	22.75%	15.37%	6.56%	9.83%	-5.63%	28.38%	4.58	1.02	90.00%
<i>Ret36</i>	27.72%	24.88%	9.19%	6.36%	-1.37%	29.10%	4.36	0.98	80.00%

Table IV
Sensitivity Analysis

The table shows the sensitivity analysis of the market-to-value ratio for a sample of firms included in the S&P 100 Index as of March 2014. The sample covers the period 1990-2013. P/V is the market-to-value ratio (i.e., the market value of equity divided by the equity value estimated by the model). The median absolute error is the median value of the absolute difference between the equity value estimated by the model and the market value of equity (in percent).

<i>Growth Rate</i>	<i>Equity Risk Premium</i>	<i>Median P/V</i>	<i>Median Absolute Error</i>
<i>Panel A: Base Case</i>			
3%	6%	1.01	33.15%
<i>Panel B: Change in Equity Risk Premium</i>			
3%	4%	0.64	62.05%
3%	5%	0.83	41.39%
3%	7%	1.18	33.60%
3%	8%	1.36	35.51%
<i>Panel C: Change in Growth Rate</i>			
1%	6%	1.44	39.31%
2%	6%	1.23	34.17%
4%	6%	0.75	45.33%
5%	6%	0.52	93.76%